(3hours)   [Total marks: 80]

N.B. (1) Question No. 1 is compulsory.
     (2) Answer **any Three** from remaining
     (3) Figures to the right indicate full marks.

1. (a) Find Laplace transform of $e^{-4t} \sin ht \sin t$.   5

   (b) Does there exist an analytic function whose real part is $x^3 - 3x^2 y - y^3$. Give
       justification.   5

   (c) Show that $\{\cos x , \cos 2x , \cos 3x ,\dots\dots\}$ is a set of orthogonal functions over
       an interval $(-\pi , \pi)$.   5

   (d) Evaluate $\int_0^{2+i} z^2 dz$ along the line joining the point $z_1 = 0$ and $z_2 = 2 + i$.   5

2. (a) Obtain the Taylor's and Laurent series which represent the function,

       $f(z) = \dfrac{1}{(z+1)(z+3)}$ valid in the regions,

       (i) $\left| z \right| < 1$ (ii) $1 < \left| z \right| < 3$ (iii) $\left| z \right| > 3$ 6

   (b) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the
       points $w = 0, i, \infty$.   6

   (c) Using Laplace transform, solve the differential equation :

       $\dfrac{d^2 x}{dt^2} + 4x = t$ with $x(0) = 1, \quad x'(0) = -2$ 8

3. (a) Solve $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial u}{\partial t} = 0$ by Bender –Schmidt method, given

       $u(0,t) = 0, u(x,0) = x(4-x), u(4,t) = 0$, assuming $h = 1$, find u upto $t=5$.   6

   (b) Using convolution theorem find the inverse Laplace transform of

       $\dfrac{s}{(s^2 + 1)(s^2 + 4)}$.   6

   (c) Determine the solution of one-dimensional heat equation $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$
       under boundary condition $u(0,t) = u(l,t) = 0, \ u(x,0) = x$, l being the
       length of rod.   8

[TURN OVER]

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4. (a) Using Residue theorem, evaluate, $\int\limits_{0}^{2\pi} \dfrac{d\theta}{5 + 3\sin\theta}$.   6

   (b) Find the inverse Laplace transform of the following:

$$\dfrac{s^2 + 2s + 3}{\left(s^2 + 2s + 2\right)\left(s^2 + 2s + 5\right)}$$ 6

   (c) Obtain Half Range Sine Series of $f(x) = x(\pi - x)$ in $(0, \pi)$.

      Hence, evaluate $-\sum\limits_{m=0}^{\infty} \dfrac{(-1)^m}{(2m+1)^3}$.

      8

5. (a) If $f(x) = e^{-3x}$, $-1 < x < 1$. Obtain Complex form of $f(x)$ in $(-1, 1)$.   6

   (b) Find the orthogonal trajectory of the family of curves $3x^2 y - y^3 = c$. 6

   (c) Solve by Crank –Nicholson simplified formula $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial u}{\partial t} = 0$,

      $u(0, t) = 0$, $u(1, t) = 2t$, $u = 0$, for two time steps taking $h = 0.25$.   8

6. (a) Obtain the Fourier series for $f(x)$ where

$$f(x) = x + \dfrac{\pi}{2} \qquad -\pi < x < 0$$

$$= \dfrac{\pi}{2} - x \qquad 0 < x < \pi$$

      6

   (b) Prove that $\int\limits_{0}^{\infty} e^{-t} \dfrac{\sin^2 t}{t} dt = \dfrac{1}{4}\log 5$ 6

   (c) Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -1$. Hence, find the image of $|z| \le 1$ onto the w-plane.   8

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