S.E - IV Sem - Chem

19/5/16

Applied Maths - IV

10/5/2016

SE/IV/CHEM/CBGS/AM-IV

Q.P. Code : 573002

(30)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **ONE** is **compulsory.**
(2) Solve any **THREE** Questions out of remaining **FIVE.**
(3) **Figures** to the right indicate **full marks.**
(4) Write the sub -questions of main question collectively together.

1. (a) Obtain the Fourier expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0\le x\le 2\pi$ **5**

(b) Evaluate $\int_c \bar{F}.d\bar{r}$ where, $\bar{F}=3xi+(2xz-y)j+Zk$ from (0, 0, 0) to (2, 1, 3) along the line joining the two points. **5**

(c) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = (3x^2-1)/2$ are orthogonal over (–1,1). **5**

(d) Find the Fourier Transform of $f(x)=e^{-x^2/2}$ **5**

2. (a) Evaluate by Green's Theorem $\oint_C (e^{2x}-xy^2)dx+(ye^x+y^2)dy$ where C is the closed curve bounded by $y^2 = x$ and $x^2 = y$. **6**

(b) Obtain half range sine series for f(x) when $f(x)=\begin{cases} x & 0<x<\pi/2 \\ \pi-x & \pi/2<x<\pi \end{cases}$ **7**

Hence, Find the sum of $\sum_{(2n-1)} \frac{1}{n^4}$

(c) Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t}=C^2\frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0,t) = 0$ $u(l, t) = 0$ and $u(x, 0) = x$ $(0 < x < l)$ l being the length of the rod. **7**

3. (a) Find the Fourier Integral representation of $f(x)=\begin{cases} 0 & x<0 \\ 1/2 & x=0 \\ e^{-x} & x>0 \end{cases}$ **6**

(b) Obtain complex form of Fourier Series for
$f(x) = \text{Cosh } 3x + \text{Sinh } 3x$ in (-3,3) **7**

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(c) Apply Stoke's Theorem to evaluate $\int_C [(x+2y)dx+(x-z)dy+(y-z)dz]$ where 7

C is the boundary of the triangle with vertics (2,0,0) (0,3,0) and (0,0,6) oriented in the counter clockwise direction.

4. (a) Prove that $\bar{F}=(6xy^2-2z^3)i+(6x^2y+2yz)j+(y^2-6z^2x)k$ is a conservative field. Find the scalar potential ϕ such that $\nabla\phi=\bar{F}$ .Hence or otherwise find the work done by $\bar{F}$ in displacing a particle from A(1,0,2) to B (0,1,1) along the straight line AB. 6

(b) Find Fourier integral representation for 7

$$f(x)= \begin{cases} 1-x^2 & \text{for } |x|\le 1 \\ 0 & \text{for } |x|>1 \end{cases}$$

(c) A rod of length 30cms has its ends A and B kept at 20°C and 80°C respectively untill steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept 80°. Find the resulting temperature function u(x,t) tanking x = 0 at A. 7

5. (a) Find the Fourier Sine transform of $\frac{e^{-ax}}{x}$ 6

(b) Find the Fourier Series for periodic function $f(x)=\begin{cases} -\pi & -\pi<x<0 \\ x & 0<x<\pi \end{cases}$ 7

(c) Evaluate $\iint_S \bar{F}\cdot d\bar{s}$ where $\bar{F}=4xi-2y^2j+z^2k$ and S is the region bounded by 7

$y^2=4x$, $x=1$ $z=0$ $z=3$

6. (a) Show that the set of functions $\sin\left(\frac{\pi x}{2L}\right)$, $\sin\left(\frac{3\pi x}{2L}\right)$, $\sin\left(\frac{5\pi x}{2L}\right)$..... is orthogonal over (0, L) 6

(b) Find half range cosine series for $f(x) = x, 0 < x < 2$ using parseval's indentity 7

deduce that $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots$

(c) Evaluate the surface integral $\iint_S (yz\hat{i}-zx\hat{j}+xy\hat{k})\cdot d\bar{s}$ where s is the surface of 7

the sphere $x^2+y^2+z^2=a^2$ in the first octant.
