S.E-IV sem - Chem 21/5/15

Applied maths-IV

CHEM/IV/CBGS/AM-IV

30

QP Code : 3029

(3 Hours) [Total Marks : 100

N.B.: (1)Question No 1 is compulsory

(2)Attempt any three questions out of the remaining five questions

(3)Non Programmable calculator is allowed

Q.1) a) Evaluate by Stokes' Theorem $\oint_C (e^x dx + 2y dy - dz)$ where C is the curve bounded by $x^2+y^2=4$, z=2 [5]

b) Show that the set of functions {sin(2n+1)x},n=0,1,2,.....is orthogonal in the interval $\left[0,\frac{\pi}{2}\right]$.Hence construct corresponding orthonormal set of functions. [5]

c) Find the Fourier sine and cosine transforms of $f(x)=x^{m-1}$.. [5]

d) For what values of x and y the given partial differential equation is hyperbolic ,parabolic ,or elliptic$(y+1)\frac{\partial^2 u}{\partial x^2}+2x\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=x+y$ [5]

Q.2) a) Find the Fourier series of $f(x)=x|x|$ in the interval (-1,1) [6]

b) Find the Fourier transform of $f(x)=1, \quad |x|<a$

$= 0, \quad |x|>a$

Hence find the value of $\int_0^\infty \frac{\sin x}{x}dx$. [6]

c) Verify Green's Theorem for $\oint_C (y-\sin x)dx+\cos x dy$ where C is the plane triangle bounded by the lines y=0, $x=\frac{\pi}{2}, y=\frac{2x}{\pi}$. [8]

Q3) a) If $\vec{F}=2xyz\hat{i}+(x^2z+2y)\hat{j}+(x^2y)\hat{k}$ then [6]

i) Prove that $\vec{F}$ is irrotational

ii) Find its scalar potential φ

iii) Find the work done in moving a particle under this force field from (0,1,1) to (1,2,0).

b) The vibrations of an elastic string is governed by the partial differential equation $\frac{\partial^2 u}{\partial t^2}=C^2\frac{\partial^2 u}{\partial x^2}$A string is stretched and fastened to two points l apart.Motion is started by displacing the string in the form y= $a\sin\left(\frac{\pi x}{l}\right)$from which it is released at time t= 0.Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t)=a\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$. [6]

c) Find the Fourier series of f(x) = πx ,0≤x<1;

= 0 , x=1

= π(x-2) ,1<x≤2

Hence deduce that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\ldots$ [8]

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(2)

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Q4) a) Find the complex form of Fourier series of f(x)=sinax in the interval $(-\pi,\pi)$ where a is not an integer. [6]

b) Evaluate $\iint_S x^3 dydz + x^2 ydzdx + x^2 zdxdy$ where S is the closed surface consisting of the circular cylinder $x^2+y^2=a^2$, z=0 and z=b. [6]

c) The equation of one dimensional heat flow is given by $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. [8]
A bar of 10 cm long with insulated sides has its ends A and B maintained at temperatures 50°C and 100°C respectively ,until steady-state conditions prevail. The temperature A is suddenly raised to 90°C and at the same time at B is lowered to 60°C.Find the temperature distribution in the bar at time t.

Q5) a) Evaluate by Green's theorem $\oint_C (y^3 - xy)dx + (xy + 3xy^2)dy$ where C is the bounded by the square with vertices $(0,0), \left(\frac{\pi}{2},0\right), \left(\frac{\pi}{2},\frac{\pi}{2}\right), \left(0,\frac{\pi}{2}\right)$. [6]

b) Find the Fourier sine integral of the function f(x) =x ,0<x<1 [6]

=2-x ,1<x<2

= 0, x>2

c) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0<x<\pi$, $0<y<\pi$, with conditions given: $u(0,y)=u(\pi,y)=u(x,\pi)=0, u(x,0)=\sin^2 x$. [8]

Q6) a) Using Stokes' theorem find the work done in moving a particle once around the perimeter of the triangle with vertices at (2,0,0),(0,3,0) and (0,0,6) under the force field
$\bar{F} = (x+y)\hat{i} + (2x-z)\hat{j} + (y+z)\hat{k}$. [6]

b) Find the half range sine series of $f(x) = x, 0 \le x \le 2;$
$= 4 - x, 2 \le x \le 4$ [6]

c) Find the Fourier cosine transform of $f(x)=\frac{1}{1+x^2}$. Hence derive the Fourier sine transform of $\frac{x}{1+x^2}$ [8]

JP-Con. : 9996-15.