SE (Chem) - Sem IV (CBGS). 23/5/14 (1)

AM IV

Applied Mathematics IV **QP Code : NP-19734**

(3 Hours) (32) **[Total Marks : 80**

N.B. : 1) Question No. 1 is Compulsory.

2)Attempt any Three Questions from remaining Five questions.

3) Non – programmable calculator is allowed.

1. a) Evaluate $\int \bar{F}.\ d\ \bar{r}$ where $\bar{F}$ = Cos y. i – x. sin y j and c is the curve $y=\sqrt{1-x^2}$ in the xy – plane from (1,0) to (0,1) (05)

   b) Find a Fourier series to represent $f(x) = x^2$ in $(0,2\pi)$. (05)

   c) Find the total work done in moving a particle in the force field $\bar{F}$ = 3xyi – 5zj + 10xk along $x=t^2+1$, $y=2t^2$, $z=t^3$ from t=1 and t=2 (05)

   d) Find the Fourier series for $f(x) = 1 - x^2$ in (-1, 1) (05)

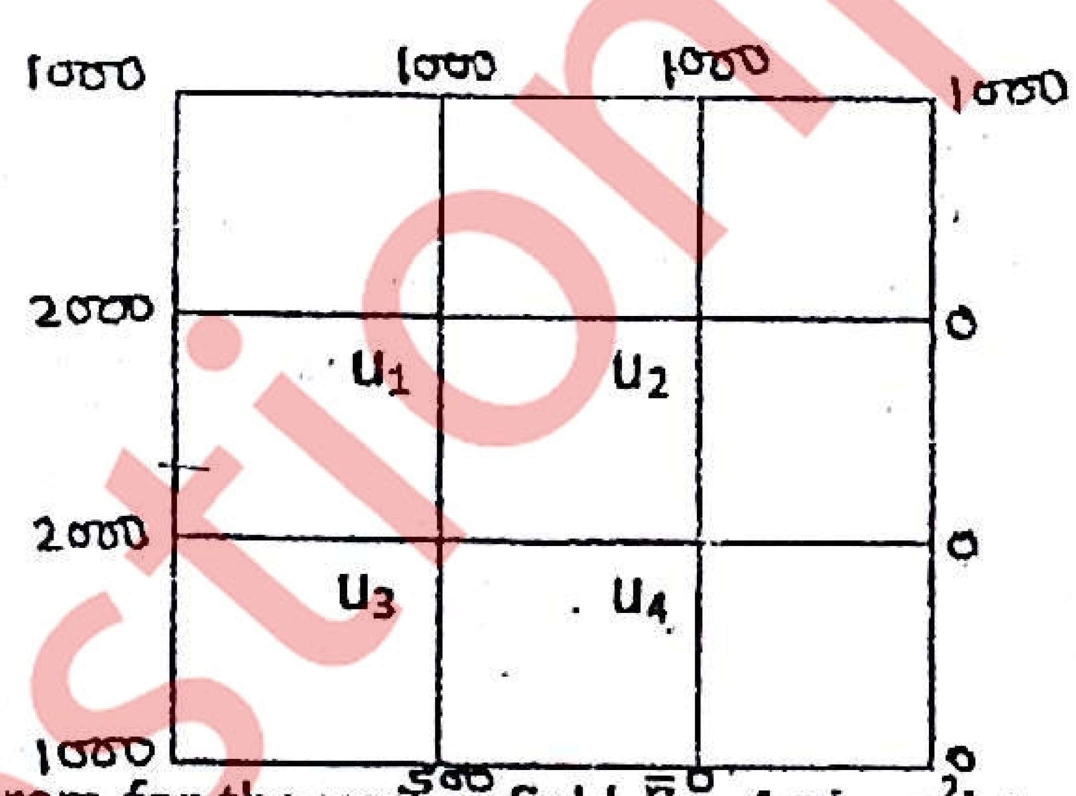
2. a) Solve the following partial differential equation.

   $3x\frac{\partial z}{\partial x} - 5y\frac{\partial z}{\partial y} = 0$ by the method of separation of variables. (06)

   b) Evaluate by Green's theorem $\int_C \bar{F}.\ d\ \bar{r}$ where $\bar{F}$ = -xy(xi – yj) and C is $r=a(1+\cos\theta)$. (07)

   c) Find a cosine series of period 2π to represent Sin x in $0 \le x \le \pi$. (07)

3. a) Solve Laplace Equation $\nabla^2 u = 0$ for the figure given below by Jacobi's method , calculate three iterations. (06)



   b) Verify Stoke's theorem for the vector field $\bar{F} = 4xzi - y^2j + yzk$ over the area in the plane z=0 bounded by x=0, y=0 and $x^2+y^2=1$. (07)

   c) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ and hence evaluate $\int_0^\infty \tan^{-1}\frac{x}{a}.\ \text{Sin}x\ dx$ (07)

4. a) Show that the set of functions $\text{Sin}(\frac{\pi x}{2L})$, $\text{Sin}(\frac{3\pi x}{2L})$, $\text{Sin}(\frac{5\pi x}{2L})$ ------ is orthogonal over (0,L) (06)

   b) Verify divergence theorem evaluate for $\bar{F}$ = 2xi + xyj + zk over the reason bounded by the cylinder $x^2 + y^2 = 4$, z=0, z=6 (07)

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c) Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions u(0,t)=0, u(l, t)=0 and u(x,0)=x, (0<x<l), l being the length of the rod. (07)

5. a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

and hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3} \right) . \cos(x/2).dx$ (06)

b) Solve the following partial differential equation $\frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} = z$ given $z(x,0) = 3e^{-5x} + 2e^{-3x}$ by the method of separation of variables. (07)

c) Show that $\bar{F} = (ye^{xy}\cos z)i + (xe^{xy}\cos z)j - (e^{xy}\sin z)k$ is irrotational and find the scalar potential for $\bar{F}$ and evaluate $\int \bar{F}.dr$ along the curve joining the points (0,0,0) and (-1, 2, π). (07)

6. a) Obtain the expansions of f(x) = $x(\pi - x)$, $0<x<\pi$ as a half – range cosine series. Hence, show that $\sum_1^\infty \frac{1}{n^4} = \frac{\pi^4}{90}$. (06)

b) Using Gauss's Divergence theorem, evaluate $\iint_S \bar{F}.d\bar{s}$ where $\bar{F} = 4xi - 2y^2j + 3z^2k$ and S is the surface $x^2 + y^2 + z^2 = a^2$, z=0, z=b. (07)

c) Solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, for the figure given below (07)