chem/IV/CBSGS/AM-IV. 1/12/16

Applied maths -IV

Q.P. Code : 573000

29

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **ONE** is **compulsory.**
(2) Solve any **THREE** Questions out of remaining **FIVE.**
(3) **Figures** to the **right** indicate **full** marks.
(4) Write the sub -questions of main question collectively together.

1. (a) Find fourier Integral of $f(x) = 1, 0 < x < 1$
$= 0, x < 0, x > 1$ **5**

(b) Evaluate $\int_c \bar{F} \times \overline{dr}$ where $\bar{F} = (2xy + z^2)i + x^2j + 3xz^2k$ along the curve **5**
$x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1)

(c) Prove that $f_1(x) = 1, f_2(x) = x, f_3(x) = (3x^2 - 1)/2$ are orthogonal over (-1,1). **5**

(d) Express the function $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ as fourier Transform. **5**

Hence, evaluate $\int_{-\infty}^{\infty} \frac{\sin s \sin sx}{s} ds$

2. (a) Obtain the Fourier expansion of **6**

$$f(x) = \left(\frac{\pi - x}{2}\right)^2 \text{ in the interval } 0 < x < 2\pi$$

(b) A tightly stretched string with fixed end points $x = 0$,& $x = L$ in the shape defined by $y = kx(L - x)$ where k is a constant is released from position of rest . Find y. **6**

(c) Prove that $\bar{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is a conservative field. Find (i) scalar potential for $\bar{F}$ (ii) the work done in moving an object in this field from (0, 1, -1) to $\left(\frac{\pi}{2}, -1, 2\right)$. **8**

3. (a) Obtain the complex form of Fourier series for **6**
$3f(x) = \cosh 3x + \sinh 3x$ in (- 3, 3)

(b) Obtain half range sine series for f(x) when **6**

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$$

Hence find the sum of $\sum_{(2n-1)}^{\infty} \frac{1}{n^4}$

(c) Verify Green's Theorem in the plane for $\int_c (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ & $y = x^2$ **8**

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chem / IV / CBSGS / APR - IV

4. (a) Find Fourier integral representation for $f(x) = 1 - x^2$ for $|x| \le 1$; $= 0$ for $|x| > 1$ **6**

(b) Show that the set of functions $\sin\frac{\pi x}{L}, \cos\frac{\pi x}{L}, \sin\frac{2\pi x}{L}, \cos\frac{2\pi x}{L}, \ldots\ldots$ **6**

Form an orthogonal set in (-l, l) and construct and orthonormal set.

(c) A rod of length 30cms has its ends A and B kept at 20^0C & 80^0C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C & kept so. Find the resulting temperature function u(x,t) taking x = 0 at A. **8**

5. (a) Find the Fourier expansion of $f(x) = 0, \quad -2 < x < -1$
$= 1+x \quad -1 < x < 0$
$= 1-x \quad 0 < x < 1$
$= 0, \quad 1 < x < 2$ **6**

(b) A rectangular metal plate with insulated surfaces is of width a and so long as compared to its breadth that it can be considered infinite in length without introducing an appreciable error if the temperature along one short edge is y = 0 given by $u(x, 0 = u\sin(\pi x/a)$ for $0 < x < a$ & other long edges x = 0 & x = a & the short edges are kept at zero degree temperature ,find the function u(x, y) describing the steady state. **6**

(c) Find the Fourier series for $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$ **8**

6. (a) Find Fourier sine Transform of $f(x) = \frac{e^{-ax}}{x}$ **6**

(b) Solve $\int_c \bar{F} \cdot d\bar{r}$ by Stokes theorem for $\bar{F} = yi + zj + xk$ over the surface $x^2 + y^2 = 1 - z$, $z > 0$. **6**

(c) Find the Fourier series for $f(x) \begin{cases} -\pi & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ **8**