S.E - IV Sem - ChemSE/IV/Chem/CBGS/AM-IV
Applied maths - IV 30/11/2015

(29)

Q.P. Code : **5374**

(3 Hours) [Total Marks :80

N.B. : (1) Question No.1 is compulsory
(2) Attempt any three questions from remaining five questions.
(3) Non Programmable calculator is allowed.

1. (a) Find a fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$ 5

(b) Find a, b, c if $\bar{F} = (x+2y+az)\,i + (bx-3y-z)\,j + (4x+cy+2z)\,k$ is irrotational. 5

(c) Find the fourier cosine transform of f(x) if 5

$$F(x)=\begin{cases}1, & 0 \le x < 1\\ 0 & x > 1\end{cases}$$

(d) Evolute $\int_A^B (ydx + xdy)$ along $y = x^2$ from A (0,0) to B(1,1). 5

2. (a) Find the Fourier expansion for $f(x) = \sqrt{1-\cos x}$ in $(0,2\pi)$. 6

Hence deduce that $\frac{1}{2} = \sum_1^\infty \frac{1}{4n^2-1}$

(b) Prove that $\bar{F} = (6xy^2 - 2z^3)\,i + (6x^2y + 2yz)\,j + (y^2 - 6z^2x)\,k$ is a conservative field. Find the scalar potential ϕ also find work done displacing particle from A (1,0,2) to B (0,1,1) 7

(c) A string is stretched and fartened to two points distance t apart. Motion is started by displacing the string in the from $y = a \sin\left(\pi x/\ell\right)$ from which it is released at time t = 0. Show that the displacement of a point at a destane x from one end at time t is given by $y(x,t) = a \sin\left(\pi x/\ell\right) \cos\left(\pi ct/\ell\right)$ 7

3. (a) Obtain half range sine series for f(x) when $f(x)=\begin{cases}x & 0 < x < \pi/2\\ \pi - x & \pi/2 < x < \pi\end{cases}$ 6

Hence, Find the sum of $\sum_{(2n-1)}^\infty \frac{i}{n^4}$

(b) Verify Green's Theorem in the plane for $\int_c (xy + y^2)dx + x^2dy$ 7

Where c is the closed curve of the region bounded by $y = x$ and $x^2 = y$

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(c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial u^2}{\partial x^2}$ under the boundary condition $u(0,t) = 0$, $u(\ell, t) = 0$ and $u(x, 0) = x$ $(0 < x < \ell)$ being the length at the rod. 7

4. (a) By using stoke's Theorem evaluate $\int_c [(x^2 + y^2)i + (x^2 - y^2)j]\, d\bar{r}$ where c is the boundary of the region enclosed by circles $x^2 + y^2 = 4$, $x^2 + y^2 = 16$ 6

(b) Show that the set of functions $\sin\left(\frac{\pi x}{2L}\right)$, $\sin\left(\frac{3\pi x}{2L}\right)$, $\sin\left(\frac{5\pi x}{2L}\right)$ ..... is over (0,L). 7

(c) Find the fourier Transform at $f(x) = e^{-|x|}$ 7

5. (a) Evaluate $\iint_c \bar{F}.d\bar{s}$ where $\bar{F} = 4xi - 2y^2j + z^2k$ and s is the region bounded by $y^2 = 4x$, $x = 1$, $z = 0$, $z = 3$ 6

(b) Find the fourier integral representation of 7

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & x = 0 \\ e^{-x} & x > 0 \end{cases}$$

(c) Obtain the complex from of fourier series for $f(x) = e^{ax}$ in $(-\ell, \ell)$ 7

6. (a) Evaluate by stoke's Theorem $\int_c (xydx + xy^2dy)$ where c is the square in the xy- plane with vertices (1,0), (0,1) (-1,0) and (0,-1) 6

(b) Find the sine transform of $\frac{1}{\sqrt{x}}$. 7

(c) Explain $f(x) = \begin{cases} kx & 0 < x < \ell/2 \\ 0 & \ell/2 < x < \ell \end{cases}$ in to half range cosine series. 7

Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$
