AM-IV

SE/Chem/KT/Rev

27/11/14

2.30 – 5.30

Applied Maths

Q.P. Code :12449

(29)

(3 Hours) [Total Marks : 80

N.B. : (1) Question No.1 is compulsory.
(2) Attempt any three out of the remaining ~~five~~ Four (04) questions.
(3) Now programmable calculator is allowed.

1. (a) Find the Fouries expansion of $f(x) = x^2$, $-\pi \le x \le \pi$ and hence, prove that $\frac{\pi^2}{6} = \sum_1^{\infty} \frac{1}{n^2}$. 5

(b) Evaluate $\int_c \overline{F} \times d\overline{r}$ where $\overline{F} = (2xy + z^2)i + x^2 j + 3xz^2 k$ along the curve $x = t$, $y = t^2$, $z = t^3$ from (0, 0, 0) to (1, 1, 1). 5

(c) Find the Fouries Transform of $f(x) = e^{-x^2/2}$. 5

(d) Find the circulation of $\overline{F}$ round the curve C where $\overline{F} = yi + zj + xk$ and C is the circle $x^2 + y^2 = a^2$ $z = 0$ 5

2. (a) Obtain half range Sine series for f(x) when : 6

$$\overline{F}(x) = \begin{cases} x & 0 < x < (\pi/2) \\ \pi - x & (\pi/2) < x < \pi \end{cases}$$

Hence, find the sum of $\sum_{2n-1}^{\infty} \frac{1}{n^4}$

(b) Show that the set of functions {sinx, Sin3x, Sin5x ....} n = 0, 1, 2 .... is orthogonal over [0,π/2]. Hence constract orthonormal set of functions. 7

(c) Find the area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ by using Green's Theorem. 7

3. (a) Find the Fourier cosine integral of the following function : 6

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & x > 2 \end{cases}$$

(b) Prove that $\int_A^B \left(2xy^3 - y^2\cos x\right)dx + \left(1 - 2y\sin x + 3x^2y^2\right)dy = \frac{\pi^2}{4}$ along arc $2x = \pi y^2$ from A(0, 0) to B($\pi/2$, 1) 7

(c) Find the Fourier expansion of $f(x) = 4 - x^2$ in the interval (0, 2). Hence prov
$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} ..... = \frac{\pi^2}{6}$

4. (a) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(-l, l)$ 6

(b) Using stoke's theorem formula $\int_c x^2dx + xy\,dy$ and C is boundary of the rectangle $x = 0,\ y = 0,\ x = a\ y = b$. 7

(c) A tightly stretched string with fixed end point $x = 0$ and $x = l$, in the shape defined by $y = kx(l - x)$ where K is constant is released from this position of rest. Find $y(x, t)$ the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. 7

5. (a) Using Gauss's Divergence theorem for $\overline{F} = \left(4xi - 2y^2j + z^2k\right)$ taken over the region bounded by $x^2 + y^2 = 4,\ z = 0,\ z = 3$. 6

(b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^4} = 0$ for $0 < x < \pi, 0 < y < \pi$ with conditions given : $u(0, y) = u(\pi, y) = u(x, \pi) = 0$, $u(x, 0) = \sin^2 x$ 7

(c) Show that $\overline{F} = \left(2xy + z^3\right)i + x^2j + 3z^2xk$ is a conservative field. Find its scalar potential and also work done in moving a particle from (1, -2, 1) to (3, 1, 4) 7