Q.P.Code: 38769

Time: 3 Hours **Marks: 80**

N.B

1. Q. No.1 is **compulsory.**
2. Answer any **four** out of remaining six questions.
3. Figures to the right indicate full marks.
4. Use of statistical tables is permitted.
5. Write the sub –questions of main question collectively together.

1. a) Using the Newton Raphson method find the root of $x^3 - 5x - 11 = 0$, **5**

b) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = (3x^2 - 1)/2$ are orthogonal over (-1,1). **5**

c) Determine the nature of the poles & find sum of residues at each pole, $(z) = \frac{z}{az^2+bz+c}$. **5**

d) Find the maximum or minimum of the function, **5**

$$z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 10x_2 - 14x_3 + 103 .$$

2. a) Find Fourier series for $f(x) = 2x - x^2$ in (0, 3). **6**

b) A tightly stretched string with fixed end points $x = 0$,& $x = L$ in the shape defined by $y = kx(L - x)$ where k is a constant is released from position of rest find y. **6**

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3rd & 3/8th rule. Also find the errors. **8**

3. a) Find the Fourier Integral representation of **6**

$$f(x) = e^{ax}, x \le 0$$
$$= e^{-ax}, \; x \ge 0.$$

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+4)(x^2+9)} dx$ using contour integration. **6**

c) A rod of length L has its ends A & B kept at 0^0C & 100^0C resp. until steady state conditions prevail. If the temperature at A is raised to 25^0C and that of B is reduced to 75^0C & kept so , find the temperature u(x, t) at a distance x from A & at time t. **8**

4. a) Find the missing terms in the following table. **6**

x:	1	2	3	4	5	6	7	8
y:	2	4	8	…..	32	…..	128	256.

b) Using Lagrange's multipliers, solve the NLPP. **6**

Optimize $z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$.

Subject to $x_1 + x_2 + x_3 = 10$,

$x_1, x_2, x_3 \geq 0$

c) Find the Fourier series for f(x) = $\begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$ **8**

5. **a)** Find all possible Laurent's series expansions of the function , $f(z) = \frac{1}{z(z+1)(z-2))}$ about z = 0 indicating the region of convergence in each case. **6**

b) A rectangular metal plate with insulated surfaces is of width a and so long as compared to its breadth that it can be considered infinite in length without introducing an appreciable error if the temperature along one short edge is y=0 given by $u(x,0) = u_0 \sin(\pi x/a)$ for 0 < x < a & other long edges x = 0 & x = a & the short edges are kept at zero degree temperature ,find the function u(x, y) describing the steady state. **6**

c) Obtain the complex form of Fourier series for $f(x) = 2x - x^2$ in (0, 2). **8**

6. **a)** Evaluate the integral $\int_C \frac{e^{z^2}}{(z+1)^4} dz$, where C is the | z – 1 | = 3 using cauchy's integral formala. **6**

b) Obtain half range sine series for $f(x) = x(2-x) in$ (0 ,2). **6**

c) Using K-T Conditions solve

$Maximize\ Z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$, **8**

Subject to $x_1 + x_2 \leq 10$, $x_2 \leq 8$, x_1, $x_2 \geq 0$.

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