S.E. chem. Choice Base Sem-IV AM-IV 8/5/2018

Q. P. Code: 38205

Time: 3 hours

Marks: 80

N.B.: (1) Question No 1 is compulsory

(2) Attempt any three questions out of the remaining five questions

(3) Non Programmable calculator is allowed

Q.1)

a) Find the value of the integral $\int_C (x+y)dx + x^2ydy$ along $y=x^2$ having (0,0) and (3,9) end points . [5]

b) Find the Fourier Series representing by $f(x)=x$, $0<x<2\pi$. [5]

c) Find the Fourier transforms of $f(x)=1-x^2$, $|x|<1$; 0 for $|x|>1$ [5]

d) Prove that $\left(\frac{\Delta^2}{E}\right)e^x.\frac{E(e^x)}{\Delta^2 e^x} = e^x$ the interval of differencing being h. [5]

Q.2)

a) Prove that $\frac{1}{2} - x = \frac{1}{\pi}\sum_1^\infty \frac{1}{n}\sin\frac{2n\pi x}{l}$,$0 < x < l$ [6]

b) Using the method of Lagrange's multipliers solve the following NLPP
Optimize $z=4x_1+8x_2-x_1^2-x_2^2$ subject to $x_1+x_2=4$, $x_1,x_2 \geq 0$ [6]

c) Expand $f(x) = \frac{1}{z^2(z-1)(z+1)}$ about z=0 indicating the region of convergence. [8]

Q.3)

a) If f(1)=4, f(2)=4,f(7)=5 and f(8)=4 ; find f(5) using Lagrange's Interpolation formula [6]

b) The vibrations of an elastic string is governed by the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$. The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0)=2(\sin x+\sin 3x)$. Find the deflection u(x,t) of the vibrating string for t>0. [6]

c) Use the Kuhn Tucker conditions to solve the following NLPP [8]
Maximize $z=2x_1+3x_2-x_1^2-2x_2^2$
subject to $x_1+3x_2 \leq 6$,
$5x_1+2x_2 \leq 10$, $x_1,x_2 \geq 0$

Q4)

a) Prove that $x(\pi - x) = \frac{\pi^2}{6} - \left[\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \cdots\ldots\right]$ in the interval $(0,\pi)$ [6]

b) Evaluate the integral using Cauchy's Integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z| = \frac{3}{2}$ [6]

c) Find the solution of one dimensional heat flow is given by $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ for which $u(0,t) = u(l,t), u(x,0) = \sin\frac{\pi x}{l}$. (0, l) [8]
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Q5)

a) If $y_0 = -8$, $y_1 = -6$, $y_2 = 22$, $y_3 = 148$, $y_4 = 492$, $y_5 = 122$ find y_6. [6]

b) Find the complex form of the Fourier series for $f(x) = e^{ax}$ over the interval $(-l, l)$ [6]

c) The diameter of a semicircular plate of radius a is kept at 0°C and the temperature at the semicircular boundary is T°C. Find the steady state temperature in the plate. [8]

Q6)

a) Using method of separation of variable solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ [6]

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$. [6]

c) (i) Use appropriate difference formula to find t when p=84 from the following data [4]

p	60	70	80	90
t	226	250	276	304

(ii) Express $f(x)=x^4-12x^3+42x^2-30x+9$ in terms of factorial polynomials. [4]

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