4-Dec-2019 1T00524 - S.E.(Chemical Engineering)(SEM-IV)(Choice Based) / 40301 - Applied Mathematics-IV
79161

[3 Hours] **[Marks : 80]**

Please check whether you have got the right question paper.

N.B:
1. Question No. 1 is compulsory.
2. Attempt any three questions from remaining five questions.
3. Figures to the right side indicate full marks.
4. Non-programmable calculator is allowed.

1. (a) Show that $\int_c \log z \, dz = 2\pi i$, where C is the unit circle in the z-plane. **05**

(b) Find the fourier series for $f(x) = 1-x^2$ in $(-1, 1)$ **05**

(c) Prove that $\left(\frac{E^4-1}{\Delta}\right) y_0 = y_0 + y_1 + y_2 + y_3$ **05**

(d) Show that the set of functions sin (2n+1) x, n = 0, 1, 2, ... is orthogonal on $\left(0, \pi/2\right)$. **05**

2. (a) Find the Fourier expansion for $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$. **06**

(b) Find the relative maximum or minimum of the function. **06**

$z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 10x_2 - 14x_3 + 103$

(c) Obtain Taylor's or Laurent's series for the function **08**

$f(z) = \frac{1}{(1+z^2)(z+2)}$ for (i) $1 < |z| < 2$ (ii) $|z| > 2$

3. (a) Find the missing entries in the following table : **06**

X	0	1	2	3	4	5
Y	1	----	11	19	----	41

(b) Solve the partial differential equation **06**

$3x\frac{\partial z}{\partial x} - 5y\frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

(c) Using the Kuhn-Tucker conditions, solve the following NLPP. **08**

Maximise $z = x_1^2 + x_2^2$

Subject to $x_1 + x_2 - 4 \le 0$

$2x_1 + x_2 - 5 \le 0$

$x_1, x_2 \ge 0$

4. (a) Obtain half-range cosine series of $f(x) = lx - x^2$, $0 < x < l$ **06**

(b) Evaluate $\oint_c \frac{\sin^6 z}{\left(z-\pi/2\right)^3} dz$ where c is $|z| = 2$ **06**

(c) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form of $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time t = 0. Show *l* that the displacement of a point at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\pi x/l\right)\cos\left(\pi ct/l\right)$ **08**

5. (a) If f(1) = 5 , f(3) = 9, f(5) = 13 , f(7) = 15 find f(2) using Lagrange's interpolation formula. **06**

(b) Obtain complex form of Fourier Series for $f(x) = e^{ax}$ in (0, a) **06**

(c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0,t) = 0, u(l,t) = 0, u(x,0) = x, (0 < x < l)$ l being length of the rod. **08**

6. (a) Using Cauchy's residue theorem evaluate $\int_c \frac{z^2}{(z-1)^2(z-2)} dz$ where c is the circle $|z|=2.5$ **06**

(b) Using the method of Lagrange's multipliers, solve the following N.L.P.P. **06**

Optimise $z = 2x_1 + 6x_2 - x_1^2 - x_2^2 + 14$

subject to $x_1 + x_2 = 4$

$x_1, x_2 \geq 0$

(c) From following data find f (6) using Newton's divided differences. **08**

x:	3	7	9	10
f(x):	168	120	72	63
