

Q.P. Code : 4904

(2 Hours)

[Total Marks : 80

N.B.: (1) Question no 1 is compulsory.

(2) Attempt any three questions out of the remaining five questions

(3) Figures to right indicate full marks.

(4) Assume any suitable data whenever required and justify the same.

1.

- a) Determine the value of k such that $w = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{kx}{y}$ is analytic. [5]
- b) Find the Laplace Transform of $e^{-t} \cos 2t \sin t$ [5]
- c) Calculate the mean and standard deviation from the following data [5]

Size of item	6	7	8	9	10	11	12
frequency	3	6	9	13	8	5	4

- d) Find the image of $|z| = 2$ under the transformation $w = z + 3 + 2i$ [5]

2.

- a) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [6]
- b) Find the orthogonal trajectories of the family of curves $x^2y - xy^3 = \text{constant}$ [6]
- c) The following marks have been obtained by a class of students in statistics [8]

Paper I	80	45	55	56	58	60	65	68	70	75	85
Paper II	81	56	50	48	60	62	64	65	70	74	90

Compute the coefficient of correlation for the above data.

3.

- a) Find Laplace Transform of $(1+2t-3t^2+4t^3)H(t-2)$ [6]
- b) A hospital switch board receives an average of 4 emergency calls in a 10 minutes interval. What is the probability that (i) there are atleast 2 emergency calls, (ii) there are exactly 3 emergency call in an interval of 10 minutes? [6]
- c) If $f(z) = u + iv$ is analytic and $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ [8]

find $f(z)$ in terms of z .

4.

- a) Find inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$ [6]
- b) Show that $u = y^3 - 3x^2y$ is harmonic. Find its harmonic conjugate and corresponding analytic function. [6]
- c) Reduce to diagonal form the following symmetric matrix A by congruent

transformation and find the rank, index and signature where $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$ [8]

[TURN OVER]

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5.

a) - Using Laplace transform evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$ [6]

b) Find the bilinear transformation which maps the points 1, -1, 2 onto the points 0, 2, -i [6]

c) Use the method of Lagrangian multipliers to solve the following problem.

$$\text{Minimize } Z = 6x_1 + 8x_2 - x_1^2 - x_2^2$$

$$\text{Such that } 4x_1 + 3x_2 = 16$$

$$3x_1 + 5x_2 = 15, \quad x_1, x_2 \geq 0$$

[8]

6.

a) Verify Cayley Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and hence evaluate

$$2A^4 - 5A^3 - 7A + 6I.$$

[6]

b) Evaluate $\int_C \frac{\sin z}{4z^2 - 3iz} dz$, C consists of the boundaries of the squares with vertices

$\pm 3, \pm 3i$ (anticlockwise) and $\pm 1, \pm i$ (clockwise)

[6]

c) Using Kuhn-Tucker conditions Minimize $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to the constraints $x_1 + 3x_2 \leq 6, 5x_1 + 2x_2 \leq 10, x_1, x_2 \geq 0$

[8]