Final

SE (Chem / ~~Biotech~~) (R3) Sem III (CBGS)

AM III (Rev)

Applied Mathematics III

QP Code : NP-18655

(3 Hours) (33) [Total Marks : 80

N.B. : (1) Questions No. 1 is compulsory.
(2) Solve any three out of the remaining five questions.
(3) Use of statistical table is permitted.

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1. (a) Find the Laplace Transform of the following $\frac{\cos\sqrt{t}}{\sqrt{t}}$ 5

(b) Verify Cayley - Hamilton Theorem for the matrix A and hence find A^{-1}. 5

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(c) Find the constants a, b, c, d, e if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic. 5

(d) The probability that a man aged 60 will live upto 70 is 0.65. What is the probability that out of 10 such men now at 60 at least 7 will live upto 70? 5

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2. (a) Prove that $\int_0^\infty \left(\frac{\sin 2t + \sin 3t}{te^t}\right) dt = \frac{3\pi}{4}$ 6

(b) Find the bilinear transformation which maps z = 2, 1, 0 onto w = 1, 0, i. 6

(c) Reduce the following quadratic form 8

$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$

to normal form through congruent transformations. Also find its rank, singnature and value class.

3. (a) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)\, dz$, along (i) the straight line joining (1-i) to (2 +i) (ii) $x = t + 1, y = 2t^2 - 1$ a parabola. 6

(b) Find the two equations of the lines of regression from the following data. 6

x :	1	2	3	4	5	6	7
y :	5	9	8	10	11	9	11

Aiso estimate the value of y for x = 8

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(c) Find the inverse Laplace transform of the following 8

(i) $\frac{3s+1}{(s+1)(s^2+2)}$ (ii) $\frac{5s^2+8s-1}{(s+3)(s^2+1)}$

4. (a) Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2 percent of the bulbs are defective. (Given $e^{-4} = 0.0183$) 6

(b) Find the coefficient of correlation between x and y for the following data. 6

x :	62,	64,	65,	69,	70,	71,	72,	74
y :	126,	125,	139,	145,	165,	152,	180,	208

(c) Find the eigen values and eigen vectors corresponding to the following matrix. 8

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

5. (a) Using the method of Lagrange's multipliers to solve the following N.L.P.P 6
optimise $Z = 4x_1 + 8x_2 - x_1^2 - x_2^2$
subject to $x_1 + x_2 = 4$, $x_1, x_2 \geq 0$

(b) The marks obtained by students in a college are normally distributed with mean 65 and variance 25. If 3 students are selected at random from this college what is the probability that at least one of them would have scored more than 75 marks? 6

(c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$ 8

6. (a) Using convolution theorem find the inverse Laplace transform of the following. 6

(i) $\frac{1}{(s-2)^4 (s+3)}$ (ii) $\frac{(s+3)^2}{(s^2+6s+5)^2}$

(b) Reduce the following quadratic form 6
$6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_1x_2 + 18x_1x_3 + 4x_2x_3$
to diagonal form through congruent transformations.

(c) Using the Kuhn - Tucker conditions solve the following N.L.P.P. 8

Maximise $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to $x_1 + x_2 \leq 2$

$2x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$
