

- N.B. (1) Questions No.1 is compulsory.
 (2) Attempt any three questions out of the remaining five questions.
 (3) Figures to the right indicate full marks.

1. (a) Find the Laplace Transform of $\frac{\cos \sqrt{t}}{\sqrt{t}}$. 5

(b) Verify Cayley-Hamilton theorem for the matrix A & hence find A^{-1} 5

$$\text{where } A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

(c) Find k such that $\frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{kx}{y}$ is analytic. -1 5

(d) The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 such men now at 60 at least 7 will live up to 70? 5

0.51626

2. (a) Find inverse Laplace Transform by using Convolution theorem 6

$$\frac{1}{s^2(s+a)^2}.$$

(b) If $f(z) = u + iv$ is analytic & $u + v = \frac{2 \sin 2x}{e^{2x} + e^{-2y} - 2 \cos 2x}$. Find $f(z)$. 6

(c) Find a matrix P that diagonalises the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ and 8

determine $P^{-1}AP$.

3. (a) Find inverse Laplace Transform of $\frac{(s^2 + 2s + 3)}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$. 6

(b) Find the Bilinear transformation which maps the points $z = -1, 1, \infty$ onto the points $w = -i, -1, i$. 6

(c) Using the Kunh - Tucker conditions solve the following N.L.P.P. 8

$$\text{Maximise : } z = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$$

$$\text{Subject to : } x_1 + x_2 \leq 2$$

$$2x_1 + 3x_2 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

4. (a) Evaluate $\int_{1-i}^{2+i} (2x+iy+1) dz$ along the curve $x = t+1$ & $y = 2t^2 - 1$. 6

- (b) Calculate the coefficient of correlation from the following data : 6

x	62	64	65	69	70	71	72	74
y	126	125	139	145	165	152	180	208

- (c) Reduce the following quadratic form $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$ to normal form through congruent transformation. Also find its rank, signature & value class. 8

5. (a) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$. 6

- (b) The marks obtained by the students in a college are normally distributed with mean 65 & variance 25. If three students are selected at random from this college what is the probability that at least one of them would have scored more than 75 marks? 6

- (c) Find the orthogonal matrix which diagonalises the real symmetric 8

matrix where $A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$.

6. (a) Using Lagrangian multiplier method solve the following N.L.P.P. 6

Optimise : $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to : $x_1 + x_2 = 4, x_1, x_2 \geq 0$

- (b) Find the eigen values & eigen vectors for $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$. 6

- (c) Find inverse Laplace Transform of 8

(i) $\frac{e^{4-3s}}{(s+4)^{5/2}}$

(ii) $\tan^{-1}\left(\frac{s+a}{b}\right)$.