

## Applied maths-III

26

(3 hours)

Total Marks:80

N.B: (1) Question no.1 is compulsory.

(2) Attempt any three questions from remaining five questions.

(3) Figures to the right indicate full marks.

(4) Assume suitable data if necessary.

1.(a) Find the Laplace Transform of  $\cos t \cos 2t \cos 3t$ . (5)

(b) If  $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$ , find  $A^7 - 9A^2 + I$ . (5)

(c) Show that  $\oint_C \log z \, dz = 2\pi i$ , where  $C$  is the unit circle  $|z|=1$ . (5)

(d) The incidence of an occupational disease in an industry is such that workers have 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will catch the disease? (5)

2.(a) Find an analytic function whose real part is  $e^{-x}(y \cos y - x \sin y)$ . (6)

(b) Evaluate  $\int_0^{\infty} e^{-t} \int_0^t \frac{\sin u}{u} \, du \, dt$ . (6)

(c) Find the orthogonal matrix that will diagonalise the matrix  $A = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$ . (8)

3.(a) Find the inverse Laplace Transform of  $\frac{s+4}{(s^2-1)(s+1)}$ . (6)

(b) Check whether the matrix  $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$  is derogatory or not. (6)

(c) Using Kuhn-Tucker conditions solve the following NLPP (8)  
 Maximize  $Z = 2x_1 + x_2 - x_1^2$   
 subject to  $2x_1 + 3x_2 \leq 6$ ;  $2x_1 + x_2 \leq 4$ ;  $x_1, x_2 \geq 0$ .

Turn Over

2

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4.(a) Find the bilinear transformation which maps the points  $z=2, i, -2$  onto the points  $w=1, i, -1$ . (6)

(b) Find the orthogonal trajectory of the family of curves given by  $e^x \cos y - xy = c$ . (6)

(c) Solve the following NLPP using Lagrange's multipliers method

Optimise  $Z = 4x_1 + 8x_2 - x_1^2 - x_2^2$  (8)  
subject to  $x_1 + x_2 = 4; x_1, x_2 \geq 0$ .

5.(a) Find the Eigen values and Eigen vectors of  $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ . (6)

(b) Evaluate  $\oint_C \frac{z+2}{z^3-2z^2} dz$  where C is the circle  $|z-2-i|=2$ . (6)

(c) Find the inverse Laplace Transform of (i)  $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$

(ii)  $\frac{(s+1)e^{-s}}{(s^2+s+1)}$ . (8)

6. (a) Find the poles and calculate the residues at them for  $f(z) = \frac{z}{(z-1)(z+2)^2}$ . (6)

(b) From the following data calculate Spearman's rank correlation coefficient between X and Y

X: 36 56 20 42 33 44 50 15 60

Y: 50 35 70 58 75 60 45 80 38. (6)

(c) Reduce the following quadratic form to canonical form. Also find its rank and signature

$x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx$ . (8)