

Subs: Applied Mathematics III

SE (Chemical & Biotech) sem III (CBSSGS)

26 NOV 2014

Sub: A M III

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QP Code: 14580

(3 Hours)

[Total Marks : 80

- N.B. (1) Question No. 1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Non-programmable calculator is allowed.

1. (a) Find Laplace transform of $e^{-4t} \sinh t \sin t$. 5
(b) Find the eigen values and eigen vectors corresponding to the following matrix:— 5

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

- (c) Evaluate $\int \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ along the curve $z = t^2 + it$. 5
(d) Show that the map of the real axis z plane is a circle under the transformation 5

$$w = \frac{2}{z+i}. \text{ Find its centre and radius.}$$

2. (a) Evaluate $\int_0^{\infty} e^{-t} \int_0^t \frac{\sin u}{u} du dt$. 6

- (b) Find the orthogonal matrix that will diagonalise the matrix. 7

$$A = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$$

- (c) If $v = e^x \sin y$, prove that the v is harmonic function. Also find the corresponding harmonic conjugate function and analytic function. 7

3. (a) Find inverse Laplace transform of $\tan^{-1} \left(\frac{2}{s^2} \right)$. 6

- (b) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming 7

matrix and the diagonal matrix.

- (c) Find the bilinear transformation which map the points 1, $-i$, 2 on z plane onto 0, 2, $-i$ respectively of w plane. 7

4. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{(2 + \cos \theta)^2}$.

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(b) Find the inverse Laplace transform by convolution theorem of $\frac{1}{(s-2)^4 \cdot (s+3)}$.

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(c) The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is $1/4$. What is the probability of 4 successes in 6 independent trials?

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5. (a) Calculate the correlation coefficient from the following data:—

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$$X : 23, 27, 28, 29, 30, 31, 33, 35, 36, 39$$

$$Y : 18, 22, 23, 24, 25, 26, 28, 29, 30, 32$$

(b) Evaluate $\int_c \frac{\sin^6 z}{\left[z - \left(\frac{\pi}{2}\right)\right]^3} dz$ where c is circle $|z| = 2$.

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(c) Find the sum of the residues at singular points of $f(z) = \frac{z}{(z-1)^2 (z^2-1)}$.

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6. (a) Find the Laplace transform of $e^{-t} \cos t \cdot H(t - \pi)$.

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(b) Using the method of Lagrangian multipliers solve the following non-linear programming problem:—

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$$\text{Maximize } Z = 6x_1 + 8x_2 - x_1^2 - x_2^2$$

$$\text{Subject to } 4x_1 + 3x_2 = 16$$

$$3x_1 + 5x_2 = 15 \quad x_1, x_2 \geq 0$$

(c) Using the Kuhn-Tucker conditions, Solve the following:—

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$$\text{N.L.P.P Maximize } Z = 2x_1 + x_2 - x_1^2$$

$$\text{Subject to } 2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4 \quad x_1, x_2 \geq 0.$$