 S.E. (Sem III) CBSGS (Chem)

29/11/13

Con. 9645-13. AM-III Applied Maths-III

GX-12086

(24)

(3 Hours) [Total Marks : ~~100~~ 80

N.B. : (1) Question No. 1 is compulsory.
(2) Attempt any ~~four~~ Three questions from remaining.

1. (a) Find Laplace transform of $\sin\sqrt{t}$. 5

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ 5

(c) Evaluate $\int_0^{1+2i} z^2 dz$ along the curve $2x^2 = y$. 5

(d) Find the mapping of the y-axis under the transformaiton $w = \frac{1}{Kz+1}$, where K is real. 5

2. (a) Evaluate $\int_0^{\infty} t\left(\frac{\sin t}{e^t}\right)^2 dt$. 6

(b) Find the orthogonal matrix which will diagonalise the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ 7

(c) Find the imaginary part of the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. Also verify that V is harmonic. 7

3. (a) Find inverse Laplace transform of $\frac{5s^2 - 15s - 11}{(s+1)(s-2)^2}$. 6

(b) Find the characteristic equation of the matrix A and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + 1$ 7

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(c) Find the bilinear transformation which maps the Points 2, i, – 2 on to the points 1, i, – 1. 7

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4. (a) Evaluate $\int_0^{\pi} \frac{d\theta}{3 + 2\cos\theta}$. 6

(b) Find inverse Laplace transform by Convolution theorem of $\frac{1}{(s+3)(s^2+2s+2)}$. 7

(c) Obtain the rank correlation coefficient from the following data : 7

X : 10, 12, 18, 18, 15, 40
Y : 12, 18, 25, 25, 50, 25

5. (a) Show that the matrix : 7

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

is diagonalisable. Find the transforming matrix and the diagonal matrix.

(b) Evaluate $\int_C \frac{z+1}{z^3 - 2z^2} dz$ where C is : 7

(i) the circle $|z| = 1$
(ii) the circle $|z - 2 - i| = 2$

(c) Determine the pole of the function $f(z) = \frac{z^2}{(z-1)^2 (z+2)}$ and also find the residue at each pole. 6

6. (a) Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five ? 6

(b) Using the method of Lanrange's multipliers solve following N.L.P.P. 7

Optimise $z = x_1^2 + 5x_2^2$
Subject to $x_1 + 5x_2 = 7$
$x_1, x_2 \geq 0$

(c) Use the Kutin-Tucker conditions to solve the following N.L.P.P. 7

Maximise $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$
Subject to $2x_1 + 5x_2 \leq 98$
$x_1, x_2 \geq 0$

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