

(3 hours)

Total Marks 80

N.B. (1) Question no. 1 is compulsory

(2) Attempt any **three** questions from remaining **five** questions.(3) **Figures** to the **right** indicate **full marks**

(4) Assume suitable data if necessary.

1. (a) Find the Laplace Transformation of $e^{-2t} \sin 3t \cos 2t$ (5)(b) If $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ then find the Eigen value of $4A^{-1} + 3A + 2I$ (5)

(c) A random variable X has following probability function (5)

X	1	2	3	4	5	6	7
P[X=x]	k	2k	3k	k^2	$k^2 + k$	$2k^2$	$4k^2$

Find (i) value of k (ii) $P[X < 5]$ (iii) $P[X > 5]$ (iv) $P[0 < X < 6]$ (d) Find P such that $\frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \left[\frac{Px}{y} \right]$ is analytic. (5)2. (a) Prove that $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$ (6)

(b) The marks obtained by students in a college are normally distributed with mean 65 and variance 25. If 3 students are selected at random from this college, what is the probability that at least one of them would have scored more than 75 marks. (6)

(c) Find the analytic function $f(z) = u + iv$ s.t. $u - v = \frac{\sin x + \cos x - e^{-y}}{2 \cos x - e^y - e^{-y}}$ when $f\left[\frac{\pi}{2}\right] = 0$. (8)3. (a) A transmission channel has a per digit error probability $p = 0.01$. Calculate the Probability of more than one error in 10 received digit using (i) Binomial distribution (ii) Poisson distribution. (6)(b) Find the inverse Laplace Transformations of (i) $\frac{3+2s+s^2}{s^3}$ (ii) $\frac{s}{(s-1)(s-2)(s-3)}$ (6)(c) Show that matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable. Find diagonal form D and diagonalising matrix M. (8)

4. (a) Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five. (6)

(b) A random sample of 50 items gives mean 6.2 and standard deviation 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% LOS. (6)

(c) (i) Find the image of $|z - 3i| = 3$ under mapping $w = \frac{1}{z}$ (6)

(ii) Find the fixed points of $w = \frac{3z-4}{z-1}$. (2)

5. (a) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (6)

(b) Theory predicts that proportion of beans in the four groups A,B,C,D should be 9: 3: 3: 1 in an experiment among 1600 beans, the numbers in the four groups 882, 313, 287 and 118. Does the experiment results supports the theory. (6)

(c) Solve the differential equation $(D^2 - 3D + 2)y = 4 e^{2t}$ given that $y(0) = -3, y'(0) = 5$. (8)

6. (a) Using convolution theorem find inverse Laplace transformation of $\frac{s^2}{(s^2+4)(s^2+25)}$ (6)

(b) Calculate the correlation of coefficient from the following data: (6)

X	23	27	28	29	30	31	33	35	36	39
Y	18	22	23	24	25	26	28	29	30	32

(c) Reduce the following quadratic form to canonical form. Also find its rank index and signature. $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$. (8)