

(3 hours)

N.B: (1) Question no.1 is compulsory.

(2) Attempt any three questions from remaining five questions.

(3) Figures to the right indicate full marks.

(4) Assume suitable data if necessary.

1. (a) Find the Laplace Transform of $e^{-2t} \frac{\sin 2t \cosh t}{t}$. (5)

(b) If $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, Verify Cayley-Hamilton Theorem for A and hence find A^{-1} and $A^3 - 5A^2$. (5)

(c) If the Probability Density function is given by $f(x) = \begin{cases} kx^2(1-x^3) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$ (5)

Find (i) k (ii) $P(0 < x < \frac{1}{2})$. (5)

(d) Show that the map of the real axis (in z plane) is a circle under the transformation

$$w = \frac{2}{z+i} \quad (5)$$

2. (a) Find an analytic function whose real part is $e^{-x}(y \cos y - x \sin y)$. (6)

(b) Evaluate $\int_0^{\infty} e^{-t} (t \int_0^1 e^{-4u} \cos u du) dt$. (6)

(c) Find the mean and standard deviation of a normal distribution of marks in an examination where 58% of the candidates obtained marks below 75, 4% got above 80 and rest between 75 and 80. (For a S.N.V the area under the curve between $z = \pm 0.2$ is 0.16 and between $z = \pm 1.8$ is .92). (8)

3 (a) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D (6)

and diagonalising matrix M. (6)

(b) If the probability that an individual suffers a bad reaction from a particular injection is 0.001, determine the probability that out of 2000 individuals (i) exactly three (ii) more than two individuals will suffer a bad reaction. (6)

Turn Over

(c) Find the inverse Laplace Transform of $\frac{s}{s^4 + 8s^2 + 16}$ using Convolution Theorem. (8) 6

4.(a) The probability of a man hitting the target is $1/4$. (i) If he fire seven times, what is the probability of him hitting the target at least twice (ii) How many times must he fire so that the probability of him hitting the target at least once is greater than $2/3$? (6)

(b) Weights in kgs. of 10 students are 38, 40, 45, 53, 47, 43, 55, 48, 52, 49. Can we say that the variance of the normal distribution from which the above sample is drawn is 20kg? $\chi^2 = 16.99$ (9 degrees of freedom at 5% LoS). (6)

(c) Find the bilinear transformation which maps the points $z = 2, i, -2$ onto the points $w = 1, i, -1$ (8)

5. (a) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (6) 4

(b) The average marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether boys perform better than the girls. ($z_{\alpha} = 2.326$) (6) 3

(c) Solve $(D^2 - 2D - 8)y = 4$, $y(0) = 0$; $y'(0) = 1$. (8) 6

6. (a) Find inverse Laplace Transform of $\frac{3s+7}{s^2-2s-3}$. (6)

(b) From the following data calculate Spearman's rank correlation coefficient between X and Y

X: 36 56 20 42 33 44 50 15 60

Y: 50 35 70 58 75 60 45 80 38.

(6)

(c) Reduce the following quadratic form to canonical form. Also find its rank and signature

$$x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx.$$

(8)