S.E – IV Sem – Biotech 19/5/16

Applied Maths – IV

9/05/2016

(24)

SE/IV/CBGS/BT/AM-IV

QP Code : 568200

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. 1 is compulsory.
(2) Answer any three questions from the remaining.
(3) Figures to the right indicate full marks.

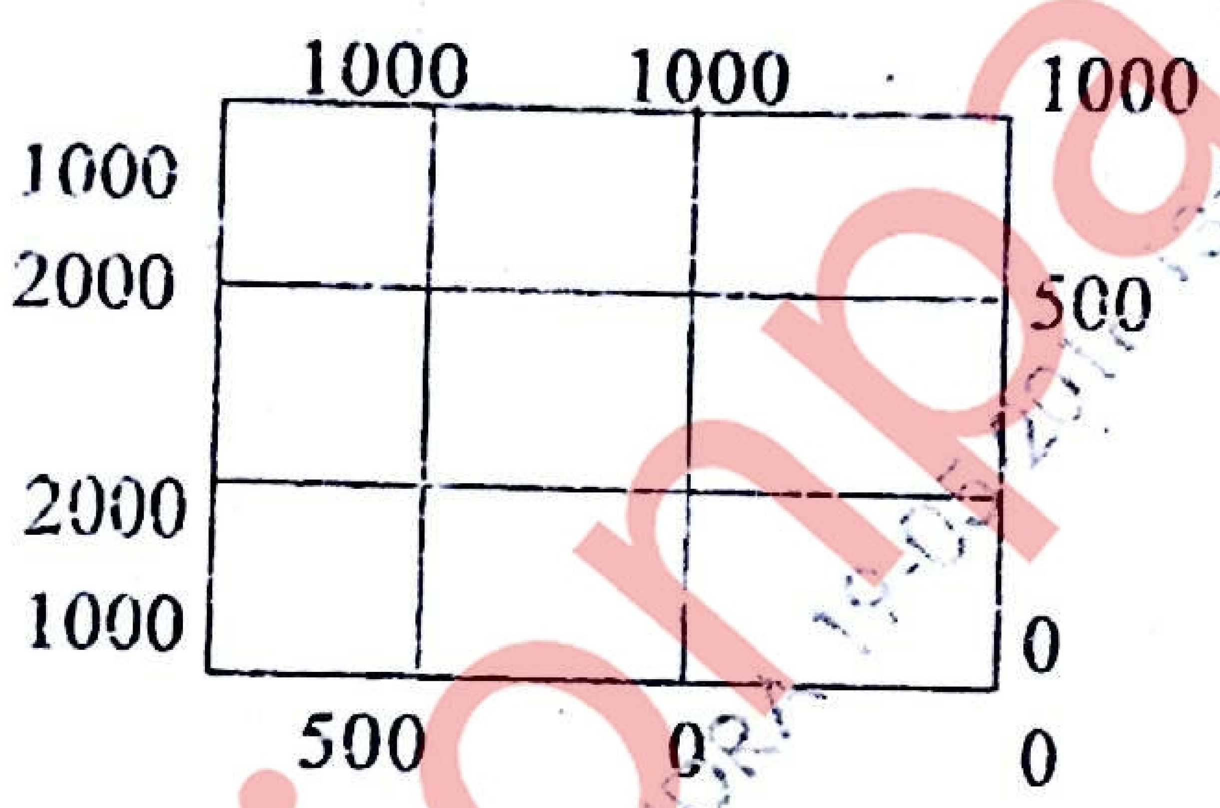
1. (a) Find the Fourier Transform of $f(x)=e^{-|x|}$

(b) Evaluate $\int_c \overline{F}.d\overline{r}$ where $\overline{F}=2xi+(xz-y)j+2zk$ from $0(0,0,0)$ to $P(3,1,2)$ along the line OP. **5**

(c) Obtain half range sine series in $(0,\pi)$ for $f(x)=x(\pi-x)$ **5**

(d) Sove $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for the following data by successive iterations. **5**

(upto 2 iterations).



2. (a) Evaluate by Green's Therorem $\int \overline{F}.d\overline{r}$ where $F=-xy(xi-yj)$ and C is $r=a(1+\cos\theta)$ **6**

(b) Obtain complex form of Fourier series for $f(x)=\cosh 2x+\sin h2x$ in (-2, 2). **6**

(c) A rod of length 'l' has its ends A and B kept at $0^{\circ}C$ and $100^{\circ}C$ respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to $0^{\circ}C$ and kept so while that of A is maintained. Find the temperature u(x,t) at a distance x from A and at time 't'. **8**

3. (a) Show that the set of functions **6**

$$1,\sin\frac{\pi x}{L},\cos\frac{\pi x}{L},\sin\frac{2\pi x}{L},\cos\frac{2\pi x}{L}........$$

form an orthogonal set is (-L, L) and construct an orthonormal set.

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(b) Express the function $f(x) = \begin{cases} \sin x & ; \ |x| < \pi \\ 0 & ; \ |x| > \pi \end{cases}$ 6

as Fourier sine integral and evaluate

$$\int_0^{\infty} \frac{\sin \omega x \sin \pi\omega}{1-\omega^2} \, d\omega$$

(c) Evaluate $\iint_S \bar{F} \cdot d\bar{s}$ where $\bar{F} = 4xi - 2y^2 j + z^2 k$ and S is the region bounded by $y^2 = 4x$, $x = 1$, $z = 0$, $z = 3$ using Gauss's Divergence Theorem. 8

4. (a) A tightly stretched string with fixed end point $x = 0$ and $x = \ell$, in the shape defined by $y = Kx(\ell - x)$ where K is a constant is released from this position of rest. Find $y(x,t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = \frac{c^2 \partial^2 y}{\partial x^2}$ 6

(b) Find the Fourier expansion of $f(x) = 2x - x^2$, $0 \le x \le 3$. 6

(c) If the vector field $\bar{F}$ is irrotational find the constants a, b, c where $\bar{F}$ is given by $\bar{F} = (x+2y+az)\, i + (bx - 3y - z)\, j + (4x + cy + 2z)\, k$. Find the Scalar potential for $\bar{F}$, and work done in moving a particle in this field from (1, 2, -4) to (3, 3, 2) along the straight line joining these points. 8

5. (a) A long rectangular plate of width 'a' cms with insulated surface has its temperature 'u' equal to zero on both the long sides and one short side so that $u(0, y) = 0$, $u(a, y) = 0$, $u(x, \infty) = 0$ and $u(x, 0) = kx$. Find the temperature $u(x,y)$ at any point of the plate in the steady-state. 6

(b) Find the Fourier cosine transform of
$f(x) = e^{-x} + e^{-2x} \quad ; \ x > 0$ 6

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(c) By using Stoke's Theorem, evaluate 8

$$\iint_s \left[\left(x^2+y^2\right)i+\left(x^2+y^2\right)j\right].d\bar{r}$$

Where C is boundary of the region enclosed by circles $x^2+y^2 = 4$, $x^2+y^2=16$

6. (a) Evaluate $\int_A^B (3x^2y-2xy)dx+(x^3-x^2)dy$ along $y^2=2x^3$ from A(0,0) and B (2, 4). 6

(b) Obtain Fourier series for 6

$$f(x)=x+\frac{\pi}{2}, -\pi<x<0$$

$$\frac{\pi}{2}-x ; 0<x<\pi$$

Hence deduce that

$$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......$$

(c) Expanel $f(x)=x\sin x$ in the interval $o \le x \le 2\pi$ 8

Deduce that $\sum_{n=2}^{\infty}\frac{1}{n^2-1}=\frac{3}{4}$