S.E - IV Sem Biotech. 2/5/15
Applied maths - IV
BT/IV/CBGS/AM-IV

(24)

QP Code : 3665

(3 Hours) [Total Marks :80

N.B. : (1) Questions No. 1 is compulsory.
(2) Attempt any **three** questions from the remaining
(3) Figures to the **right** indicate full marks.

1 (a) Obtain half range cosine series for $f(x) = x(\pi - x),\ 0 < x < \pi$. 5

(b) Find the total work done in moving a particle in the force field 5

$\bar{F} = 3xy\,i - 5z\,j + 10x\,k$ along $x = t^2 + 1$,
$y = 2t^2$, $z = t^3$ from $t=1$ and $t=2$.

(c) Prove that 5

$$\int_A^B (2xy^3 - y^2\cos x)\,dx + (1 - 2y\sin x + 3x^2y^2)\,dy = \frac{\pi}{4}$$

(d) Show that the set of functions cosx, cos 2x, cos3x ... is a set of orthogonal functions over $(-\pi, \pi)$, Hence, construct a set of orthonormal functions. 5

2. (a) Find the Fourier expansion of $f(x) = 2x - x^2$, $0 \le x \le 3$ whose period is 3. 6

(b) Evaluate $\iint_S (\nabla \times \bar{F})\cdot d\bar{s}$ where 6

$\bar{F} = (2y^2+3z^2-x^2)\,i + (2z^2+3x^2-y^2)\,j + (2x^2+3y^2-z^2)k$
over the part of the sphere $x^2+y^2+z^2-2ax+az=o$ cut off by the plane $z=0$

(c) A string is stretched between $x=0$ and $x=l$ and both ends given a displacement $y = a\sin pt$ perpendicular to the string. If the string satisfies the differntiel equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$, show that the oscillation of the string are given by 8

$$y = a\sec\frac{p\ell}{2c}\cos\left(\frac{px}{c} - \frac{p\ell}{2c}\right)\sin pt.$$

3. (a) A tightly stretched string with fixed ends $x=o$ and $x=\ell$ and initially in a position $y = a\sin^3(\pi x/\ell)$ is released from the position of rest. Find the displacement of any point at any time if y, the vertical displacement satisfies the equation 6

$$\frac{\partial^2 y}{\partial t^2} = C^2\frac{\partial^2 y}{\partial x^2}.$$

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3. (b) Using Fourier cosine integral, prove that 6

$$e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \left(\frac{\omega^2+2}{\omega^2+4}\right)\cos \omega x \, d\omega$$

(c) Solve the equation $\frac{\partial u}{\partial t} = \frac{K\partial^2 u}{\partial^2 x^2}$ for the conduction of heat along a rod of length 8

ℓ subject to the following conditions.

(i) u is not infinitely for $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = \ell$ for any time t.

(iii) $u = \ell x - x^2$ for $t = 0$ between $x = 0$ and $x = \ell$.

4. (a) Evaluate $\iint_S x^2 \, dydz + y^2 \, dz\,dx + 2z(xy - x - y)dx\,dy$ 6

where S is the surface of the cube bounded by

$x = 0, x = 1, y = 1, z = 0, z = 1.$

(b) Obtain Fourier series of xcosx in $(-\pi, \pi)$. 6

(c) Expand $f(x) = x \sin x$ in the interval $0 \le x \le 2\pi$. 8

Hence deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$.

5. (a) Find complex form of Fourier series for cos ax, where a is not an integer is $(-\pi, \pi)$. 6

(b) A rectangular plate with insulted surface has width of a'cms and so long compared to its width that it may be considered infinite is length without introducing an appreciable error. If the two long edges x=0 and x = a as well as the one short edge are kept at 0°C and the temperature of the other short edge y = 0 is given by 6

$u = kx$ for $0 \le x \le a/2$

$= k(a - x)$ for $a/2 \le x \le a$

Find the temperature u (x, y) at any point (x, y) of the plate.

(c) Show that 8

$\bar{F} = (2xyz^2)\,i + (x^2 z^2 + z \cos yz)\,j + (2x^2 yz + y \cos yz)\,k$ is conservative. Find the scalar potential ϕ such that $\bar{F} = \nabla\phi$ and hence, find the work done by $\bar{F}$ in displacing a particle from A (0,0,1) TO B (1, π / 4, 2) along the straight line AB.

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6. (a) Find the Fourier transform of 6

$$f(x) = 1 + \frac{x}{a} \; ; \; -a < x < 0$$

$$1 - \frac{x}{a} \; ; \; 0 < x < a$$

(b) Using Green's Theorem evaluate $\int_C \overline{F}.d\overline{r}$ where C is the curve enclosing the region bounded by $y^2 = yax$, $x = a$ in the plane $z = o$ and $\overline{F} = (2x^2y + 3z^2)\,i + (x^2+4yz)\,j + (2y^2 + 6xz)\,k$. 6

(c) Find the Fourier expansion of 8
$f(x) = x + x^2 \; ; \; -\pi \leq x \leq \pi$
Hence deduce that

(i) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots..$

(ii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.$

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