SE · Sem · IV (Bio · Tech)
SE (BioTech) - Sem IV (CBGS) 23/5/14

AM IV
Applied mathematics - IV

QP Code : **NP-19740**

(3 Hours) (26) [Total Marks : 80

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **three** questions from the **remaining** questions.
(3) **Figures to right** indicate **full** marks.

1. (a) Find half range cosine series for $f(x) = x,\ 0 < x < 2$. 5
(b) Find the total work done in moving a particle in the force field $\overline{F} = 3\,xyi - 5\,zj + 10\,xk$ along $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$. 5
(c) Evaluate $\int_C \overline{F}\cdot d\overline{r}$ where
$\overline{F} = \cos y\, i - x \sin y\, j$ and C is the curve
$y = \sqrt{1-x^2}$ in xy-plane from (1, 0) to (0, 1). 5
(d) Show that the set of function cos x, cos 2x, cos 3x . . . . is a set of orthogonal function over $(-\pi, \pi)$. Hence, construct a set of Orthonormal function. 5

2. (a) Find the Fourier expansion of —
$$f(x) = \begin{cases} 2, & -2 \le x \le 0 \\ x, & 0 < x \le 2 \end{cases}$$ 6
(b) Evaluate $\iint_S \left(\nabla \times \overline{F}\right)\cdot d\overline{s}$ where
$\overline{F} = (2y^2 + 3z^2 - x^2)i + (2z^2 + 3x^2 - y^2)j + (2x^2 + 3y^2 - z^2)k$
over the part of the sphere $x^2 + y^2 + z^2 - 2\,ax + az = 0$ cut-off by the plane $z = 0$. 6
(c) An elastic string is stretched between two points at a distance 'ℓ' apart. In its equilibrium position at a point at a distance 'a' $(a < \ell)$ from one end is displaced through a distance 'b' transversely and then released from this position. Obtain y(x, t) the vertical displacement if y satisfies the equation — 8
$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

3. (a) A string is stretched and fastened to two points 'ℓ' apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{\ell}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time it is given by 6
$$y(x, t) = a \sin \frac{\pi x}{\ell} \cos \frac{\pi c t}{\ell}.$$

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(b) Find the Fourier sine transform of e^{-x}, $x \geq 0$ and hence deduce that — 6

$$\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2} e^{-m} \; ; \; m > 0.$$

(c) Find Fourier series for f(x) in (0, 2π). 8

$$f(x) = \begin{cases} x, & 0 < x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$$

Hence deduce that —

$$\frac{\pi^2}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots..$$

4. (a) Using Gauss-Divergence Theorem, prove that — 6

$$\iint_S \left(y^2z^2 i + z^2x^2 j + z^2y^2 k\right) \cdot \overline{N}\, ds = \pi/12.$$

Where s is the point of the sphere $x^2 + y^2 + z^2 = 1$ above xy-plane.

(b) Solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ for the condition of heat along a rod of length ℓ subject to the following conditions :— 6

(i) u is not infinity for $t \to \infty$.

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = \ell$ for any time t.

(iii) $u = \ell x - x^2$ for $t = 0$ between $x = 0$ and $x = \ell$.

(c) Obtain Fourier series for $f(x) = x + \pi/2, -\pi < x < 0$ 8

$= \pi/2 - x, 0 < x < \pi$

Hence deduce that —

$$\frac{\pi^4}{46} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots..$$

5. (a) Obtain the complex form of Fourier series for f(x) = cosh ax in $(-\ell, \ell)$. 6

(b) A rectangular plate with insulated surface has width of 'a' in and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the two long edges x = 0 and x = a as well as the one short edge are kept at 0°C and the temperature of the other short edge y = 0 is given by 6

$u = kx \; ; \quad 0 \leq x \leq a/2$

$k(a - x) \; ; \quad a/2 \leq x \leq 0.$

Find the temperature u(x, y) at any point (x, y) of the plate.

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(c) Prove that $\overline{F} = (y^2 \cos x + z^3)i + (2y \sin x - y)j + (3xz^2 + 2)k$ is a conservative field. Find — **8**

(i) Scalar potential for $\overline{F}$

(ii) Work done in moving an object in this field from $(0, 1, -1)$ to $(\pi/2, -1, 2)$.

6. (a) Find the Fourier integral representation of $f(x) = e^{-|x|}$; $-\infty < x < \infty$. **6**

(b) Using Green's theorem, evaluate $\int_C \overline{F} \cdot d\overline{r}$ where C is the curve enclosing the region bounded by $y^2 = 4ax$, $x = a$ and $\overline{F} = (2x^2y + 3z^2)\, i + (x^2 + 4yz)j + (2y^2 + 6xz)k$. **6**

(c) Expand $f(x) = x \sin x$ in the interval $0 \le x \le 2\pi$. Deduce that **8**

$$\frac{3}{4} = \sum_{n=2}^{\infty} \frac{1}{n^2 - 1}.$$
