Rev 27/11/14

SE (Biotech) Sem IV [CBSGS]

Sub: AM IV 2:30-5:30

Applied maths - IV **QP Code : 12455**

(23)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Questions No. 1 is **compulsory.**

(2) Attempt any **three** questions from the remainirig five questions.

(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find half range sine series for $f(x) = x \sin x$ in $(0, \pi)$. 5

(b) Find the work dene is moving a pacticle once round the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in the plane $z = 0$ in the four field given by $\bar{F} = (3x - 2y)i + (2x + 3y)j + y^2 k$. 5

(c) Evaluate $\int_C \bar{F} \cdot \bar{dr}$ where $\bar{F} = yzi + zxj + xy k$ and C is the portion of the curve. $\bar{r} = a \cos t\, i + b \sin t\, j + ct\, k$ from $t = 0$ to $t = \pi/4$. 5

(d) Show that the set of funtion $f(r) = \sin (2n + 1)x$; $n = 0, 1, 2, 3$--- in $(0, \pi/2)$ is orthogonal. Hence construct the orthonormal set of functions. 5

2. (a) Find the Fourier series of $f(x) = \begin{cases} \pi x; & 0 \le x \le 1 \\ \pi(2 - x); & 0 \le x \le 2 \end{cases}$ 6

(b) Evaluate $\iint_S (\nabla \times \bar{F}) \cdot \bar{ds}$ where 6

$\bar{F} = (2x - y + z)\, i + (x + y - z^2) + (3x - 2y + 4z)\, k$
and S is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane $z = 9$ and open at the other end.

(c) A tightly stretched string with fixed end points $x = 0$ and $x = l$, in the shape defined by $y = kx\,(l - x)$ where k is a constant, is released from this position of rest. Find $y(x, t)$, the vertical displacement if 6

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

3. (a) An eleastic string is stretched between two points at a distance π apart. In its equillibrium position the string in the shape of the curve $f(x) = K(\sin x - \sin 2x)$. Obtain $y(x, t)$ the vertical displacement if y satisfies the equation 8

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}.$$

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(b) Find Fourier integral representation of

$$f(x) = \begin{cases} e^{ax} \; ; \; x \le 0 \\ e^{-ax} \; ; \; x \ge 0 \end{cases} \; ; \; a > 0.$$

Hence show that

$$\int_0^\infty \frac{\cos \omega x}{\omega^2 + a^2} d\omega = \frac{\pi}{2a} e^{-ax} \; ; x>0, a>0.$$

(c) Solve the equation $\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation subject to the following conditions :– 8

(i) u is finite when $t \to \infty$

(ii) $\partial u/\partial x = 0$ when $x = 0$ for all values of t.

(iii) $u = 0$ when $x = \ell$ for all values of t.

(iv) $u = u_0$ when $t = 0$ for $0 < x < \ell$.

4. (a) Evaluate $\iint_S \bar{F} \cdot d\bar{s}$ where $\bar{F} = 4xi - 2y^2 j + z^2 K$ and S is the region bounded by $y^2 = 4x$, $x = 1$, $z = 0$, $z = 3$. 6

(b) Find Fourier series for $f(x) = |\sin x|$ in $(-\pi, \pi)$. 6

(c) Find 8

$$f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$$ in $(0, 2\pi)$. Hence declare that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ...$$

5. (a) Obtain the complex form of Fourier series for $f(x) = \cosh ax$ is $(-\ell, \ell)$. 6

(b) Obtain a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ to satisfy the following conditions – 6

(i) $u \to 0$ as $y \to \infty$ for all x.

(ii) $u = 0$, if $x = 0$ for all y.

(iii) $u = 0$ if $x = \ell$ for all y

(iv) $u = \ell x - x^2$ if $y = 0$ for all values of x between 0 and ℓ.

(c) If the vector field $\bar{F}$ is irrotational find the constants a, b, c where $\bar{F}$ is given by $\bar{F} = (x + 2y + az)\, i + (bx - 3y - z)\, j + (4x + cy + 2z)\, K$ Find the scalar potential for $\bar{F}$ and also find the work done in moving a practicle in this field from (1, 2, –4) to (3, 3, 2) along the straight line joining these points. 8

6. (a) Find Fourier sine integral representation for 6

$$f(x) = \frac{e^{-ax}}{x}\ ;\ x>0.$$

(b) Evaluate by Green's Theorem 6

$$\int_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$$

where C is the boundary of the region bounded by $y = x^2$ and $y^2 = x$.

(c) Optain Fourier series for 8

$$f(r) = x + \frac{\pi}{2} \quad -\pi < x < 0$$

$$\frac{\pi}{2} - x \quad ; 0 < x < \pi$$

Hence deduce that

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...$$

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