Applied mathematics III

BT/CHEM/CBGS/ III /AM-III

Q.P. Code : **4904**

(2 Hours) [Total Marks : 80

N.B.: (1) Question no 1 is compulsory.
(2) Attempt any three questions out of the remaining five questions
(3) Figures to right indicate full marks.
(4) Assume any suitable data whenever required and justify the same.

1.

a) Determine the value of k such that $w=\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\frac{kx}{y}$ is analytic. [5]

b) Find the LaplaceTransform of $e^{-t}\cos 2t\sin t$ [5]

c) Calculate the mean and standard deviation from the following data [5]

Size of item	6	7	8	9	10	11	12
frequency	3	6	9	13	8	5	4

d) Find the image of $|z| = 2$ under the transformation $w=z+3+2i$ [5]

2.

a) Find the eigenvalues and eigenvectors of $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [6]

b) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = constant$ [6]

c) The following marks have been obtained by a class of students in statistics [8]

Paper I	80	45	55	56	58	60	65	68	70	75	85
Paper II	81	56	50	48	60	62	64	65	70	74	90

Compute the coefficient of correlation for the above data.

3.

a) Find Laplace Transform of $(1+2t-3t^2+4t^3)H(t-2)$ [6]

b) A hospital switch board receives an average of 4 emergency calls in a 10 minutes interval .What is the probability that (i) there are atleast 2 emergency calls, (ii) there are exactly 3 emergency call in an interval of 10 minutes? [6]

c) If f(z) =u+iv is analytic and $u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ [8]

find f(z) in terms of z.

4.

a) Find inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$ [6]

b) Show that $v=y^3-3x^2y$ is harmonic. Find its harmonic conjugate and corresponding analytic function. [6]

c) Reduce to diagonal form the following symmetric matrix A by congruent transformation and find the rank ,index and signature where $A=\begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$ [8]

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(2)

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5.

a) Using Laplace transform evaluate $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\,dt$ [6]

b) Find the bilinear transformation which maps the points 1,-i,2 onto the points 0,2,-i [6]

c) Use the method of Lagrangian multipliers to solve the following problem. [8]

Minimize $Z=6x_1+8x_2-x_1^2-x_2^2$

Such that $4x_1+3x_2=16$

$3x_1+5x_2=15$, $x_1,x_2,\geq 0$

6.

a) Verify Caley Hamilton Theorem for $A=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$ and hence evaluate [6]

$2A^4-5A^3-7A+6I$.

b) Evaluate $\int_C \frac{\sin z}{4z^2-8iz}dz$, C consists of the boundaries of the squares with vertices [6]

$\pm 3, \pm 3i$ (anticlockwise) and $\pm 1, \pm i$ (clockwise)

c) Using Kuhn-Tucker conditions Minimize $z=2x_1+3x_2-x_1^2-2x_2^2$ [8]

Subject to the constraints $x_1+3x_2\leq 6$, $5x_1+2x_2\leq 10$, $x_1, x_2\geq 0$