Applied maths-III

(3 hours)   Total Marks: 80

N.B: (1) Question no.1 is **compulsory**.

(2) Attempt any **three** questions from remaining **five** questions.

(3) **Figures** to the **right** indicate **full** marks.

(4) Assume suitable data if necessary.

1.(a) Find the Laplace Transform of $\cos t \cos 2t \cos 3t$.   (5)

(b) If $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$, find $A^7 - 9A^2 + I$.   (5)

(c) Show that $\oint_c \log z \, dz = 2\pi i$, where C is the unit circle $|z| = 1$.   (5)

(d) The incidence of an occupational disease in an industry is such that workers have 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will catch the disease?   (5)

2.(a) Find an analytic function whose real part is $e^{-x}(y \cos y - x \sin y)$.   (6)

(b) Evaluate $\int_0^\infty e^{-t} \int_0^t \frac{\sin u}{u} \, du \, dt$.   (6)

(c) Find the orthogonal matrix that will diagonalise the matrix $A = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$.   (8)

3.(a) Find the inverse Laplace Transform of $\frac{s+4}{(s^2-1)(s+1)}$.   (6)

(b) Check whether the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory or not.   (6)

(c) Using Kuhn-Tucker conditions solve the following NLPP
Maximize $Z = 2x_1 + x_2 - x_1^2$
subject to $2x_1 + 3x_2 \le 6; 2x_1 + x_2 \le 4; x_1, x_2 \ge 0$.   (8)

Turn Over

4.(a)Find the bilinear transformation which maps the points z= 2,i,-2 onto the points w=1,i,-1. (6)

(b)Find the orthogonal trajectory of the family of curves given by $e^x \cos y - xy = c$. (6)

(c)Solve the following NLPP using Lagrange's multipliers method

$Optimise\ Z = 4x_1 + 8x_2 - x_1^2 - x_2^2$ (8)
$subject\ to\ x_1 + x_2 = 4; x_1, x_2 \geq 0.$

5.(a)Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (6)

(b)Evaluate $\oint_C \frac{z+2}{z^3 - 2z^2} dz$ where C is the circle $|z-2-i| = 2$. (6)

(c)Find the inverse Laplace Transform of (i) $\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$

(ii) $\frac{(s+1)e^{-s}}{(s^2 + s + 1)}$. (8)

6. (a)Find the poles and calculate the residues at them for $f(z) = \frac{z}{(z-1)(z+2)^2}$. (6)

(b)From the following data calculate Spearman's rank correlation coefficient between X and Y

X: 36 56 20 42 33 44 50 15 60

Y: 50 35 70 58 75 60 45 80 38. (6)

(c)Reduce the following quadratic form to canonical form.Also find it's rank and signature

$x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx.$ (8)