Subj: Applied Mathematics III

SE (Chemical & Biotech) Sem III (CBSGS)

26 NOV 2014

Sub: A M III

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QP Code : **14580**

(3 Hours) **[Total Marks : 80**

N.B. (1) Question No. 1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Non-programmable calculator is allowed.

1. (a) Find Laplace transform of e^{-4t} sinht sint. 5
(b) Find the eigen values and eigen vectors corresponding to the following matrix :— 5

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}.$$

(c) Evaluate $\int \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve $z = t^2 + it$. 5

(d) Show that the map of the real axis z plane is a circle under the transformation 5

$w = \dfrac{2}{z+i}$. Find its centre and radius.

2. Evaluate $\int_0^{\infty} e^{-t} \int_0^{t} \dfrac{\sin u}{u}\, du\, dt$. 6

(b) Find the orthogonal matrix that will diagonalise the matrix. 7

$$A = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}.$$

(c) If $v = e^x \sin y$, prove that the v is harmonic function. Also find the corresponding harmonic conjugate function and analytic function. 7

3. (a) Find inverse Laplace transform of $\tan^{-1}\left(\dfrac{2}{s^2}\right)$. 6

(b) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming 7
matrix and the diagonal matrix.

(c) Find the bilinear transformation which map the points 1, –i, 2 on z plane onto 0, 2, –i respectively of w plane. 7

4. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$. 6

(b) Find the inverse Laplace transform by convolution theorem of $\frac{1}{(s-2)^4 \cdot (s+3)}$. 7

(c) The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trails is 1/4. What is the probability of 4 successes in 6 independent trails ? 7

5. (a) Calculate the correlation coefficient from the following data :— 6

X : 23, 27, 28, 29, 30, 31, 33, 35, 36, 39

Y : 18, 22, 23, 24, 25, 26, 28, 29, 30, 32

(b) Evaluate $\int_C \frac{\sin^6 z}{\left[z-(\pi/2)\right]^3} dz$ where c is circle $|z| = 2$. 7

(c) Find the sum of the residues at singular points of $f(z) = \frac{z}{(z-1)^2(z^2-1)}$. 7

6. (a) Find the Laplace transform of e^{-t} cost. $H(t-\pi)$. 6

(b) Using the method of Lagrangian multipliers solve the following non-linear programming problem :— 7

Maximize $Z = 6x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to $4x_1 + 3x_2 = 16$

$3x_1 + 5x_2 = 15$ $x_1, x_2 \geq 0$

(c) Using the Kuhn-Tucker conditions, Solve the following :— 7

N.L.P.P Maximize $Z = 2x_1 + x_2 - x_1^2$

Subject to $2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$ $x_1, x_2 \geq 0$.