

- N.B. : (1) Question No. 1 is compulsory.
 (2) Attempt any three questions from remaining five questions.
 (3) Figures to the right indicate full marks.

1. (a) Find the Laplace Transform of $\sin t \sin 2t \sin 3t$. 5
 (b) Find $2A^3 - 3A^4 + A^2 - 2I$ where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. 5
 (c) Evaluate $\int |z| dz$ along the left half of $|z|=1$ from $Z = -i$ to $Z = i$. 5
 (d) The proofs of a 500 page book contain 500 misprints. Find the probability that there are atleast 4 misprints in a randomly chosen page. 5
2. (a) Find an analytic function whose imaginary part is $\cosh x \cos y + \frac{x}{x^2 + y^2}$. 6
 (b) Evaluate $\int_0^{\infty} t e^{-4t(3t)} dt$ if $L(f(t)) = \frac{1}{\sqrt{s^2 + 1}}$. 6
 (c) Using Kuhn-Tucker conditions solve the following :-
 NLPP Maximize $Z = 2x_1 + 3x_2 - x_1^2 - x_2^2$
 Subject to $x_1 + x_2 \leq 1$, $2x_1 + 3x_2 \leq 6$; $x_1, x_2 \geq 0$ 8
3. (a) Find minimal polynomial of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$. 6
 Is it derogatory matrix?
 (b) The life of Army Shoes normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? 6
 (c) Find the bilinear transformation which maps the points $Z = 1, i, -1$ onto the points $W = i, 0, -i$. Hence find the image of $|Z| < 1$ onto the W -plane. 8

4. (a) Find the inverse Laplace Transform using convolution theorem $\frac{1}{(S-4)^2(S+3)}$ 6
- (b) Find the orthogonal Trajectory of the family of curves $e^x \cos y + e^y \cos x = c$ 6
- (c) Using Lagrange's Method solve the following NLPP 8
 Optimise $Z = 3x_1^2 + x_2^2 + x_3^2$
 Subject to $x_1 + x_2 + x_3 = 2, x_1, x_2, x_3 \geq 0$

5. (a) Find Eigen values and Eigen vectors of A^2 if :-

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

- (b) Evaluate $\int_c \frac{\sin \pi z + \cos \pi z}{z^2 + z} dz; \text{cis}|Z|=4$ using Cauchy's integral formula. 6

- (c) Find inverse Laplace Transform of :- 8

(i) $\frac{1}{S} \text{Log}(1 + \frac{1}{S^2})$

(ii) $\frac{(S+1)e^S}{S^2+S+1}$

6. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using Residue theorem. 6

- (b) Find the coefficient of correlation between X and Y. 6

X	14	8	10	11	9	13	5
Y	14	9	11	13	11	12	4

- (c) Find the rank, Signature and class of the following Quadratic form by reducing it to its canonical form using congruent transformations 8

$$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_1x_3 + 2x_1x_2.$$

