

Duration : 3 hrs

Max. marks :80

1. Question no.1 is compulsory
2. Answer **any three** from remaining.
3. Figures to the right indicate marks.

1.a. Find the Laplace transform of $\sin t \sin 2t \sin 3t$ (5)

b. The proofs of a 500 page contain 500 misprints. Find the probability that at least 4 misprints in a randomly chosen page. (5)

c. check whether this matrix is $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is derogatory? (5)

d. Find the analytic function whose imaginary part is $e^x \sin y$ (5)

2a. Show that $u = \left(r + \frac{a^2}{r}\right) \cos \theta$ is harmonic. Find $v(r, \theta)$ (6)

b. Find $L\left(\sinh\left(\frac{\sqrt{3}}{2}t\right) \sin\left(\frac{t}{2}\right)\right)$ hence evaluate

$$\int_0^{\infty} e^{-t} \sinh\left(\frac{\sqrt{3}}{2}t\right) \sin\left(\frac{t}{2}\right) dt \quad (6)$$

c. Using Lagrangian multiplier, solve the N.L.P. (8)

Optimize $Z = x_1^2 + x_2^2 + x_3^2$ subject to $x_1 + x_2 + 3x_3 = 2$,

$$5x_1 + 2x_2 + x_3 = 5, \quad x_1, x_2, x_3 \geq 0$$

3.a. Find the image of the region bounded by $x=0, x=2, y=0, y=2$ in the (8)

Z plane under the transformation $w = (1+i)z$

b. Show that $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal

matrix and transforming matrix. (6)

c. find the equations of lines of regression for the data given below. (8)

x	5	6	7	8	9	10	11
y	11	14	14	15	12	17	16

4.a. Solve N.L.P.P using Kuhn tucker conditions (6)

Maximize $Z = 2x_1^2 - 7x_2^2 + 12x_1x_2$ subject to $2x_1 + 5x_2 \leq 98, x_1, x_2 \geq 0$

b. Using Convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ (6)

c. Evaluate $\int_0^\pi \frac{d\theta}{3+2\cos\theta}$ (8)

5.a Evaluate $\int_0^{1+i} z^2 dz$ along (i) the line $y = x$ (ii) the parabola $x = y^2$. (6)

Is the line integral independent of path? Explain.

b. Find the residues of $f(z) = \frac{\sin \pi z}{(z-2)(z-1)^2}$ at its poles (6)

c. Reduce the following quadratic form $q = X^T AX$ (8)

$3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form by orthogonal transformation and hence find the rank, index and signature

6. a. Evaluate $\int_C \frac{z+3}{2z^2+3z-2} dz$ where C is $|z-i|=2$ (6)

b. In 100 sets of 10 tosses of a coin in how many cases do you expect (6)

(i) 7 head and 3 tails

(ii) At least 7 heads

(8)

c. Find inverse laplace transform of by using partial fractions

$$\frac{s^2+16s-24}{s^4+20s^2-64}$$
