

Duration : 3 hrs

Max. marks :80

1. Question no.1 is compulsory
2. Answer **any three** from remaining.
3. Figures to the right indicate marks.

- 1.a. Show that $u = \left(r + \frac{a^2}{r}\right) \cos \theta$ is harmonic. Find $v(r, \theta)$ (5)
- b. Find the Laplace transform of $t^3 \cos t$ (5)
- c. If 10% of bolts are defective calculate probability that out of a sample selected at random of 7 bolts not more than one bolt will be defective. (5)
- d. check whether this matrix is $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory? (5)
- 2.a. If $f(z) = u + iv$ is an analytic function of z and $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ then find $f(z)$ given that $f(z) = \frac{\pi}{2}$ (6)
- b. Prove that $\int_0^{\infty} \frac{\sin 2t + \sin 3t}{te^t} dt = \frac{3\pi}{4}$ (6)
- c. Using Lagrangian multiplier, solve the N.L.P.P. (8)
- Optimize $Z = x_1^2 + x_2^2 + x_3^2$ subject to $x_1 + x_2 + 3x_3 = 2$,
 $5x_1 + 2x_2 + x_3 = 5, x_1, x_2, x_3 \geq 0$
- 3.a. Find the orthogonal trajectory of the family of curves given by $x^3y - xy^3$ (6)
- b. Show that matrix $A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$ is diagonalizable and find the diagonal and transforming matrix. (6)
- c. find the equations of lines of regression for the data given below. (8)

x	5	6	7	8	9	10	11
y	11	14	14	15	12	17	16

4.a. Solve N.L.P.P using Kuhn tucker conditions

Maximize $Z = 2x_1 - x_1^2 + x_2$ subject to $2x_1 + 3x_2 \leq 6, 2x_1 + x_2 \leq 6$
 $x_1, x_2, \geq 0$

(6)

b. Using Convolution theorem find $L^{-1}[\frac{s^2}{(s^2+1)(s^2+4)}]$

(6)

c. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5+4\cos\theta} d\theta$

(8)

5. a. Find image of circle $|z|=2$ under $w=-z+(3+2i)$

(6)

b. Find the residues of $f(z) = \frac{\sin \pi z}{(z-2)(z-1)^2}$ at its poles

(6)

c. Reduce the following quadratic form $q = X^T AX$

(8)

$3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form by orthogonal transformation and hence find the rank, index and signature

6. a. Evaluate $\oint_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $|z+1|=1$

(6)

b. In 100 sets of 10 tosses of a coin in how many cases do you expect

(6)

- (i) 7 head and 3 tails
- (ii) At least 7 heads

(8)

c. Find inverse laplace transform by using partial fractions

$$\frac{s^2 + 16s - 24}{s^4 + 20s^2 - 64}$$
