SE (Elect | ETRX | Bio- | EXTC) SEM-IV (CBGS)
AM-IV
May 2016 19/5/16

QP Code : 545802

(3 Hours) **[Total Marks: 80**

N.B.: (1) Question **No.1** is **compulsory**.
(2) Attempt **any Three** from the remaining.

1. (a) Find the extremal of the functional **5**

$$\int_0^1 [y'^2 + 12xy]dx \text{ subject to } y(0) = 0 \text{ and } y(1) = 1.$$

(b) Verify Cauchy - Schwartz inequality for $u = (1,2,1)$ and $v = (3,0,4)$ also find the angle between u & v. **5**

(c) If λ & X are eigen values and eigen vectors of A then prove that $\frac{1}{\lambda}$ and X are eigen values and eigen vectors of A^{-1}, provided A is non singular matrix. **5**

(d) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where $C : |z| = 2$ **5**

2. (a) Find the extremal that minimises the integral **6**

$$\int_{x_0}^{x_1} (16y^2 - y''^2) dx$$

(b) Find eigen values and eigen vectors of A^3 **6**

where $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

(c) Obtain Taylor's and two distinct Laurent's expansion of $f(z) = \frac{z-1}{z^2 - 2z - 3}$ indicating the region of convergence. **8**

[TURN OVER

3. (a) Verify Cayley-Hamilton Theorem for **6**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find } A^{-1}$$

(b) Using Cauchy Residue Theorem, evaluate **6**

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx$$

(c) Show that a closed curve 'C' of given fixed length (perimeter) which encloses maximum area is a circle. **8**

4. (a) Find an orthonormal basis for the subspace of R^3 by appling Gram-Schmidt process where S {(1,1,1), (0,1,1) (0,0,1)}. **6**

(b) Find A^{50}, where **6**

$$A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$$

(c) Reduce the following Quadratic form into canonical form & hence find its rank, index, signature and value class where, **8**

$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

5. (a) Using the Rayleigh-Ritz method, find an approximate solution for the extremal of the functional $\int_0^1 \{xy + \frac{1}{2}y'^2\}\, dx$ subject to $y(0) = y(1) = 0$. **6**

(b) Prove that $W = \{(x,y) \mid x = 3y\}$ subspace of R^2. Is $W_1 = \{(a,1,1) \mid a \text{ in } R\}$ subspace of R^3 ? **6**

[TURN OVER

(c) Prove that A is diagonizable matrix. Also find diagonal form and transforming matrix where $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ **8**

6. (a) By using Cauchy Residue Theorem, evaluate $\int_0^{2\pi} \frac{\cos^2\theta}{5+4\cos\theta} d\theta$ **6**

(b) Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where $C : |z+1+i| = 2$. **6**

(c) (i) Determine the function that gives shortest distance between two given points. **5**

(ii) Express any vector (a,b,c) in R^3 as a linear combination of v_1, v_2, v_3 where v_1, v_2, v_3 are in R^3 . **3**
