SE / Sem IV / Biomed / Choice Based 07/05/19

Paper / Subject Code: 40101 / Applied Mathematics-IV

Duration: 3 hours **Max. Marks 80**

N. B.: 1. Question No. 1 is Compulsory.
2. Attempt any 3 Questions from Question no. 2 to 6.
3. Figures to the right indicate the full Marks.
4. Statistical tables are allowed.

Que. 1 a If λ is an eigen value of square matrix A then prove that λ^n is an eigen value of matrix A^n 5

b Let X be a continuous random variable with probability density function $f(x)=kx(1-x),\ 0 \le x \le 1$. Find k and determine the number 'b' such that $P(X \le b) = P(X \ge b)$ 5

c Find a basis for the orthogonal complement of the subspace in R^3 spanned by the vectors $V_1 = (1,-1,\ 3)$, $V_2 = (5,-4,-4)$, $V_3 = (7,-6,2)$ 5

d Evaluate $\int_{-2}^{2} \frac{2z+3}{z} dz$ along the upper half of the circle $|z| = 2$ 5

Que.2. a If $A=\begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ find eigen values and eigen vectors of A^2-2A+I. 6

b In a precision bombing attach there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target. 6

c Find all Taylor and Laurent series expansions for $f(z) = \frac{z}{(z-2)(z-3)}$ about z=1 indicating the region of convergence. 8

Que.3. a Three factories A, B, and C produces 35%, 45% and 20% of the total production of an item. Out of their production 90%, 50%, and 10% are defective. Find probability that it is produced by factory A 6

b Verify Cayley-Hamilton theorem for $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} 6

c Obtain the equations of the lines of regression for the following data. Also obtain the estimate of X for Y=70. 8

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

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Que.4. a By using Cauchy's residue theorem, evaluate $\oint_C \frac{\sin \pi z + \cos \pi z}{(z-1)(z-2)} dz$ where C is $|z| = 3$ — 6

b Construct an orthonormal basis of R^3 using Gram Schmidt process to S={(3, 0, 4), (-1, 0, 7), (2, 9, 11)} — 6

c Determine whether the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable, if yes diagonalise it. — 8

Que. 5 a Show that the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory and find the minimal polynomial of the matrix. — 6

b The weekly wages of 1000 workmen are normally distributed around a mean of Rs 70 and standard deviation Rs 5. Estimate the number of workers whose weekly wages will be (i) between 65 and 75 (ii) more than 75 — 6

c Find the curve $y = f(x)$ for which $\int_{x_1}^{x_2} y\sqrt{1+y'^2}\, dx$ is minimum subject to the constraint $\int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx = l$. — 8

Que.6. a If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ show that $A^{100} = \begin{bmatrix} -299 & -300 \\ 300 & 301 \end{bmatrix}$ — 6

b Between 2 pm and 4 pm, the average number of phone calls per minute coming into a switchboard of a company is 2.5. Find the probability that during one particular minute there will be (i) no phone call at all, (ii) at least 5 calls. — 6

c By using Cauchy residue theorem, evaluate — 8

i. $\int_0^{\infty} \frac{dx}{x^2+9}$ ii. $\int_0^{2\pi} \frac{1}{5+4\cos\theta} d\theta$