E/Sem - IV/ Bio-Med /Choice Based/ AM-IV

Applied Mathematics - IV

Q.P. Code: 37604

Duration: 3 hours

(16)

Max. Marks 80

N. B.: 1. Question No. 1 is Compulsory.

2. Attempt any 3 Questions from Question no. 2 to 6.

3. Figures to the right indicate the full Marks.

4. Statistical tables are allowed.

Que. 1 a If λ is an eigen value of square matrix A then prove that λ^n is an eigen value of matrix A^n 5

b Let X be a continuous random variable with probability density function $f(x)=kx(1-x), \quad 0 \le x \le 1$. Find k and determine the number 'b' such that $P(X \le b) = P(X \ge b)$ 5

c Find a basis for the orthogonal complement of the subspace in R^3 spanned by the vectors $V_1 = (1, -1, 3)$, $V_2 = (5, -4, -4)$, $V_3 = (7, -6, 2)$ 5

d Evaluate $\int_{-2}^{2} \frac{2z+3}{z} dz$ along the upper half of the circle $|z| = 2$ 5

Que.2. a Using Rayleigh-Ritz method, solve the boundary value problem 6

$I = \int_0^1 (y'^2 - y^2 - 2xy)dx; \quad 0 \le x \le 1$. Given y(0)=0 and y(1)=0

b In a precision bombing attach there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? 6

c Find all Taylor and Laurent series expansions for $f(z) = \frac{z}{(z-2)(z-3)}$ about z=1 indicating the region of convergence. 8

Que.3. a Three factories A, B, and C produces 35%, 45% and 20% of the total production of an item. Out of their production 90%, 50%, and 10% are defective. Find probability that a selected defective item is produced by factory A 6

b Verify Cayley-Hamilton theorem for A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} 6

c Obtain the equations of the lines of regression for the following data. Also obtain the estimate of X for Y=70. 8

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

31

Que.4. a Find the extremal of the functional $\int_0^{\pi/2}(y'^2 - y^2 + 2xy)\,dx$ with $y(0)=0$ and $y\left(\frac{\pi}{2}\right)=0$.

b Construct an orthonormal basis of R^3 using Gram Schmidt process to $S=\{(3,0,4),(-1,0,7),(2,9,11)\}$

c Determine whether the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ is diagonalizable, if yes diagonalise it.

Que. 5 a Show that the matrix $A=\begin{bmatrix}5 & -6 & -6\\ -1 & 4 & 2\\ 3 & -6 & -4\end{bmatrix}$ is derogatory and find the minimal polynomial of the matrix. 6

b A random variable X has p.d.f. $\frac{1}{2^x}$, x=1,2, 3,.. Find the moment generating function and hence find mean and variance of X. 6

c The weekly wages of 1000 workmen are normally distributed around a mean of Rs 70 and standard deviation Rs 5. Estimate the number of workers whose weekly wages will be (i) between 69 and 72 (ii) more than 75 (iii) less than 63. Also estimate the lowest wages of the 100 highest paid workers. 8

Que.6. a If $A=\begin{bmatrix}2 & 3\\ -3 & -4\end{bmatrix}$ show that $A^{100}=\begin{bmatrix}-299 & -300\\ 300 & 301\end{bmatrix}$ 6

b Between 2 pm and 4 pm, the average number of phone calls per minute coming into a switchboard of a company is 2.5. Find the probability that during one particular minute there will be (i) no phone call at all, (ii) at least 5 calls. 6

c By using Cauchy residue theorem, evaluate 8

i. $\int_0^{\infty}\frac{dx}{x^2+9}$ ii. $\int_0^{2\pi}\frac{1}{5+4\cos\theta}\,d\theta$