Paper / Subject Code: 40101 / Applied Mathematics-IV

SE/Biomed/Sem IV/Choice based.

(7)

Duration: 3 hours

Q.P. Code: 37603

(14)

Max. Marks 80

N. B.: 1. Question No. 1 is Compulsory.
2. Attempt any 3 Questions from Question no. 2 to 6.
3. Figures to the right indicate the full Marks.
4. Statistical tables are allowed.

Que. 1 a If λ is an eigen value of nonsingular matrix A then prove that $\frac{|A|}{\lambda}$ is an eigen value of $adj\ A$. 5

b If the random variable X takes the values 1, 2, 3, 4 such that 2P(X=1)=3P(X=2)=P(X=3)=5P(X=4), find the probability distribution and cumulative distribution of X. 5

c Find a basis for the orthogonal complement of the subspace in R^3 spanned by the vectors $V_1 = (1, -1, 3)$, $V_2 = (5, -4, -4)$, $V_3 = (7, -6, 2)$ 5

d Evaluate the complex line Integral $\oint_C \log z\, dz$ where C is the unit circle $|z|=1$ 5

Que.2. a Using Rayleigh-Ritz method, solve the boundary value problem

$I = \int_0^1 (y'^2 - y^2 - 2xy)dx;\ 0 \le x \le 1$. Given y(0)=0 and y(1)=0 6

b Seven dice are thrown 729 times. How many times do you expect at least 4 dice to show 3 or 5? 6

c Find all Taylor and Laurent series expansions for $f(z) = \frac{z}{(z-3)(z-4)}$ about z=1 indicating the region of convergence. 8

Que.3. a Three factories A, B, and C produces 30%, 20% and 50% of the total production of an item. Out of their production 70%, 50%, and 30% are defective. Find probability that a defective item selected is produced by factory A 6

b Verify Cayley-Hamilton theorem for A= $\begin{bmatrix} 3 & 2 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} 6 c Obtain the equations of the lines of regression for the following data. Also obtain the estimate of X for Y=70. 8

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

Que.4. a Find the extremal of the functional $\int_0^{\pi/2}(y'^2 - y^2 + 2xy)\,dx$ with $y(0)=0$ and $y\left(\frac{\pi}{2}\right)=0$. 6

b Construct an orthonormal basis of R^3 using Gram Schmidt process to S={(3, 0, 4), (-1, 0, 7), (2, 9, 11)} 6

c Determine whether the matrix $A=\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable, if yes diagonalise it. 8

Que. 5 a Show that the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ is derogatory and find the minimal polynomial of the matrix. 6

b A random variable X has probability density function $\frac{1}{2^x}$, x=1, 2, 3,.. Find moment generating function and hence find mean and variance of X. 6

c Of a group of men 5% are under 60 inches height and 40% are between 60 and 65 inches. Assuming a normal distribution find the mean height and standard deviation. 8

Que.6. a If $A=\frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ find e^A and 4^A 6

b Between 2 pm and 4 pm, the average number of phone calls per minute coming into a switchboard of a company is 2.5. Find the probability that during one particular minute there will be i) no phone call at all, ii) at least 5 calls. 6

c By using Cauchy residue theorem, evaluate 8

i. $\int_0^{\infty}\frac{dx}{x^2+4}$ ii. $\int_0^{2\pi}\frac{1}{5-4\cos\theta}\,d\theta$

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