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(3 Hours)
[Revised Course]

[Total Marks : 80]

- N.B.:
- 1) Question No.1 is compulsory.
 - 2) Attempt any three from the remaining questions.
 - 3) Assume suitable data if necessary.

1. (a) Determine the constants a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$ is analytic. 5
- (b) Find a cosine series of period 2π to represent $\sin x$ in $0 \leq x \leq \pi$ 5
- (c) Evaluate by using Laplace Transformation $\int_0^\infty e^{-3x} t \cos t dt$. 5
- (d) A vector field is given by $\vec{F} = (x^2 + xy^2) i + (y^2 + x^2 y) j$. Show that \vec{F} is irrotational and find its scalar potential. Such that $\vec{F} = \nabla \phi$. 5
2. (a) Solve by using Laplace Transform $(D^2 + 2D + 5)y = e^t \sin t$, when $y(0) = 0, y'(0) = 1$. 6
- (b) Find the total work done in moving a particle in the force field $\vec{F} = 3xy i - 5z j + 10x k$ along $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ and $t = 2$. 6
- (c) Find the Fourier series of the function $f(x) = e^{-x}, 0 < x < 2\pi$ and $f(x + 2\pi) = f(x)$. Hence deduce that the value of $\sum_{n=2}^\infty \frac{(-1)^n}{n^2 + 1}$. 8
3. (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$ 6
- (b) Verify Green's theorem in the plane for $\oint (x^2 - y) dx + (2y^2 - x) dy$ 6
Around the boundary of region defined by $y = x^2$ and $y =$
the Laplace transforms of the following. 8

$$e^{-x} \int_0^t \frac{\sin u}{u} du$$

$$\text{ii) } t \sqrt{1 + \sin t}$$

OVER

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- 4 (a) If $f(x) = C_1 Q_1(x) + C_2 Q_2(x) + C_3 Q_3(x)$, where C_1, C_2, C_3 constants and Q_1, Q_2, Q_3 are orthonormal sets on (a, b) , show that

$$\int_a^b [f(x)]^2 dx = c_1^2 + c_2^2 + c_3^2.$$

- (b) If $v = e^x \sin y$, prove that v is a Harmonic function. Also find the corresponding harmonic conjugate function and analytic function.

- (c) Find inverse Laplace transforms of the following.

i) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

ii) $\frac{s+2}{s^2-4s+13}$

- 5 (a) Find the Fourier series if $f(x) = |x|$, $-k < x < k$

Hence deduce that $\sum \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$.

- (b) Define solenoidal vector. Hence prove that $\vec{F} = \frac{\vec{r} \times \vec{r}}{r^n}$ is a solenoidal vector.

- (c) Find the bilinear transformation under which $1, i, -1$ from the z -plane are mapped onto $0, 1, \infty$ of w -plane. Further show that under this transformation the unit circle in w -plane is mapped onto a straight line in the z -plane. Write the name of this line.

- 6 (a) Using Gauss's Divergence Theorem evaluate $\iint_s \vec{F} \cdot d\vec{s}$ where $\vec{F} = 2x^2 y \vec{i} - y^2 \vec{j} + 4xz^2 \vec{k}$ and s is the region bounded by $y^2 + z^2 = 9$ and $x = 2$ in the first octant.

- (b) Define bilinear transformation. And prove that in a general, a bilinear transformation maps a circle into a circle.

- (c) Prove that $\int x^{2/3} (x^{3/2}) dx = -\frac{2}{3} x^{-1/2} \Big|_{-1/3} (x^{3/2})$.