SE Sem-III (Biomed)

Applied Mathematics-III

QP Code : NP-18646

(21)

(3 Hours) [Total Marks : 80

N. B. : (1) Question No. 1 (one) is compulsory.
(2) Attempt any 3 (three) questions from the remaining questions.
(3) Assume suitable data, if necessary.

1. (a) Evaluate $\int_0^{\infty} \frac{(\cos 6t - \cos 4t)}{t}\, dt$ 5

(b) Obtain complex form of fourier series for $f(x) = e^{ax}$ in $(-1,1)$ 5

(c) Find the work done in moving a particle in a force field given by $\overline{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. 5

(d) Find the orthogonal trajectory of the curves $3x^2y + 2x^2 - y^3 - 2y^2 = \alpha$, where α is a constant. 5

2. (a) Evaluate $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$, $y(o) = 0$, $y'(o) = 0$, by Laplace transform 6

(b) Show that $J_{5/2} = \sqrt{\frac{2}{\pi x}}\left[\frac{3 - x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$ 6

(c) (i) Find the constants a, b, c so that $\overline{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + (y + 2z)\hat{k}$ is irrotational. 4

(ii) Prove that the angle between two surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2,-1,2)$ is $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ 4

3. (a) Obtain the fourier series of f(x) given by 6

$$f(x) = \begin{cases} 0 & ,-\pi \le x \le 0 \\ x^2 & 0 \le x \le \pi \end{cases}$$

(b) Find the analylic function $f(z) = u + iv$ where $u = r^2 \cos 2\theta - r\cos\theta + 2$ 6

(c) Find Laplace transform of 8

(i) $te^{-3t}\cos 2t.\cos 3t$

(ii) $\frac{d}{dt}\left[\frac{\sin 3t}{t}\right]$

4. (a) Evaluate $\int J_3(x)dx$ and Express the result in terms of J_0 and J_1 6
(b) Find half range sine series for 6
$f(x) = \pi x - x^2$ in $(0, \pi)$

Hence deduce that $\frac{\pi^3}{32} = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots$

(c) Find inverse Laplace transform of

(i) $\frac{1}{s}\tanh^{-1}(s)$ (ii) $\frac{se^{-2s}}{(s^2+2s+2)}$

5. (a) Under the transformation $w+2i = z+\frac{1}{z}$, show that the map of the circle $|z| =$ 2 is an ellipse in w-plane. 6
(b) Find half range cosine series of $f(x) = \sin x$ in $0 \le x \le \pi$. 6
Hence deduce that

$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots=\frac{1}{2}$

(c) Verify Green's theorem, for 8

$\oint_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where c is boundary of the region defined by $x=0$, $y=0$, and $x+y = 1$.

6. (a) Using convolution theorem; evaluate 6

$L^{-1}\left\{\frac{1}{(s-1)(s^2+4)}\right\}$

(b) Find the bilinear transformation which maps the points 6
$z = 1, i, -1$ onto $w = 0, 1, \infty$
(c) By using the appropriate theorem, Evaluate the following :- 8

(i) $\int \bar{F}\cdot d\bar{r}$ where $\bar{F} = (2x-y)\hat{i} - (yz^2)\hat{j} - (y^2z)\hat{k}$
and c is the boundary of the upper half of the sphere $x^2 + y^2 + z^2 = 4$

(ii) $\iint_s (9x\hat{i} + 6y\hat{j} - 10z\hat{k})\cdot d\bar{s}$ where s is
the surface of sphere with radius 2 units.
