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Applied maths - III  
(Revised course)

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Time : 3 hours

Total marks : 80

- N.B : (1) Question No.1 is compulsory.  
(2) Answer any three questions from remaining.  
(3) Assume suitable data if necessary.

Evaluate

1. (a)  $\int_0^{\pi} e^{-t} \left( \frac{\cos 3t - \cos 2t}{t} \right) dt$  05

(b) Obtain the Fourier Series expression for  $f(x) = 2x - 1$  in  $(0, 3)$  05

(c) Find the value of 'p' such that the function  $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \left( \frac{py}{x} \right)$  is analytic. 05

(d) If  $\vec{F} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$ .  
Show that  $\vec{F}$  is irrotational. Also find its scalar potential. 05

2. (a) Solve the differential equation using Laplace Transform 06

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = 3te^{-t}, \text{ given } y(0) = 4 \text{ and } y'(0) = 2$$

(b) Prove that 06

$$J_4(x) = \left( \frac{48}{x^3} - \frac{8}{x} \right) J_1(x) - \left( \frac{24}{x^2} - 1 \right) J_0(x)$$

(c) i) In what direction is the directional derivative of  $\phi = x^2 y^2 z^4$  at  $(3, -1, -2)$  maximum. Find its magnitude. 08

ii) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$   
Prove that  $\nabla r^n = nr^{n-2} \vec{r}$

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2

SE/III/CBGS/EXTC  
QP Code : 5106

3. (a) Obtain the Fourier Series expansion for the function

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \leq x \leq 0$$

$$= 1 - \frac{2x}{\pi}, 0 \leq x \leq \pi$$

06

(b) Find an analytic function  $f(z) = u + iv$  where.

06

$$u - v = \frac{x - y}{x^2 + 4xy + y^2}$$

(c) Find Laplace transform of

08

i)  $\cosh t \int_0^t e^u \sinh u$

ii)  $t\sqrt{1 + \sin t}$

4. (a) Obtain the complex form of Fourier series for  
 $f(x) = e^{\alpha x}$  in  $(-L, L)$

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(b) Prove that

$$\int x^4 J_1(x) dx = x^4 J_1(x) - 2x^3 J_2(x) + c$$

06

(c) Find

08

i)  $L^{-1} \left[ \frac{2s-1}{s^2+4s+29} \right]$

ii)  $L^{-1} \left[ \cot^{-1} \left( \frac{s+3}{2} \right) \right]$

5. (a) Find the Bilinear Transformation which maps the points  
 $1, i, -1$  of  $z$  plane onto  $0, 1, \infty$  of  $w$ -plane

06

(b) Using Convolution theorem find

06

$$L^{-1} \left[ \frac{s^2}{(s^2+4)^2} \right]$$

(3)  
3

(c) Verify Green's Theorem for  $\int_C \vec{F} \cdot d\vec{r}$  where 08

$\vec{F} = (x^2 - y^2)\hat{i} + (x+y)\hat{j}$  and C is the triangle with vertices (0,0), (1,1) and (2,1)

6. (a) Obtain half range sine series for  
 $f(x) = x, 0 \leq x \leq 2$

06

$$= 4 - x, 2 \leq x \leq 4$$

(b) Prove that the transformation

$w = \frac{1}{z+i}$  transforms the real axis of the z-plane into a circle in the w-plane.

06

(c) i) Use Stoke's Theorem to evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where

08

$\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$  and C is the rectangle in the plane  $z=0$ , bounded by  $x=0, y=0, x=a$  and  $y=b$ .

ii) Use Gauss Divergence Theorem to evaluate

$\iiint_S \vec{F} \cdot \hat{n} ds$  where  $\vec{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$  and S is the surface bounded by  $x=0, y=0, z=0$  and  $2x+2y+z=4$