

08/05/19



Max. Marks 80

Duration: 3 hours

- N. B.: 1. Question No. 1 is Compulsory.
 2. Attempt any 3 Questions from Question no. 2 to 6.
 3. Figures to the right indicate the full Marks.

- Que. 1 a Find Laplace transform of $e^{-2x} \sin 4t$ 5
 b Expand Fourier series for $f(x) = x^3$ in $(-\pi, \pi)$ 5
 c If $\vec{F}(x, y, z) = (2x + y)\mathbf{i} - 6y\mathbf{j} + az\mathbf{k}$ is Solenoidal, find a and find curl of \vec{F} . 5
 d Show that $f(z) = z^n$ is analytic, hence find $f'(z)$ 5
- Que. 2 a Find Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$ 6
 b Prove that $J_{3/2}(x) = -\sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]$ 6
 c By using Laplace transform, Solve $\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 4e^{-4x}$, where $y(0) = 0, y'(0) = 1$ 8
- Que. 3 a Evaluate $\int_0^{\infty} e^{-2x} \frac{\cos 4t - \cos 6t}{t} dt$, by using Laplace transform 6
 b Find analytic function whose real part is $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6
 c Verify Green's theorem $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 - xy)\mathbf{i} + (x^2 - y^2)\mathbf{j}$ and C is the closed curve bounded by $x^2 = 2y$ and $x=y$ 8
- Que. 4 a Show that $\vec{F} = (y^2 \cos x + z^3)\mathbf{i} + (2y \sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}$ is irrotational, hence find its scalar potential. 6
 b Find complex form of Fourier series $f(x) = \cosh 2x + \sinh 2x$ in $(-1, 1)$ 6
 c Find bilinear transformation which maps the points $2, -i, -2$ onto the points $1, i, -1$, hence find its fixed points. 8
- Que. 5 a Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (y^2 + z^2 - x^2)\mathbf{i} + (z^2 + x^2 - y^2)\mathbf{j} + (x^2 + y^2 - z^2)$ over the boundary of the surface. $x^2 + y^2 - 2ax + az = 0$. 6
 b Find inverse Laplace transform of i. $\frac{1}{s} \tan^{-1} \left(\frac{a}{s} \right)$ ii. $\frac{e^{-ks}}{(s^2 + 2s + 2)}$ 6
 c Find Fourier integral representation for $f(x) = \begin{cases} -e^{-kx} & \text{for } x < 0 \\ e^{-kx} & \text{for } x > 0 \end{cases}$ 8
 Hence prove that $\int_0^{\infty} \frac{\sin \omega x}{\omega^2 + k^2} d\omega = \frac{\pi}{2} e^{-kx}$ if $k > 0, x > 0$.

TURN OVER

- Que. 6. a Show that the set of functions $\{\sin x, \sin 3x, \sin 5x, \dots\}$ is orthogonal over $[0, 2\pi]$. Construct orthonormal set of functions.
- b Find inverse Laplace transform of $\frac{x}{(s^2+a)(s^2+b)}$ by using convolution theorem
- c Find the images of the infinite strips $i. \frac{1}{2b} < y < \frac{1}{2a}$ $ii. 0 < y < \frac{1}{2a}$ under transformation $w = \frac{1}{z}$. Show the regions graphically where $a > 0, b > 0$ and $a < b$.