Paper / Subject Code: 50501 / Applied Mathematics-III

SE / Bio med / Sem III / Choice Based.

39

Time: 3 Hours

16

Total Marks: 80

N.B. 1) Question No. 1 is compulsory.

2) Attempt any three out of remaining five Questions.

3) Figure to right indicate full mark.

Q. 1 a) Find the constant a, b, c, d ,e if $F(z) = ax^3+bxy^2+3x^2+cy^2+x+i(dx^2y-2y3+exy+y)$ is Analytic. (05)

b) Find directional derivatives of $\emptyset = 4xy^2-2yz^2$ at $[1,-2,,2]$ in the direction of $2i+3j+5k$ (05)

c) Show that the set of functions $Fn(x) = \cos nx$, n = 1,2,3 ....is orthogonal over $[-\pi,\pi]$ Hence construct orthonormal set (05)

d) Find $L[e^{-2t}\ \cos 2t.\cos 4t\ ]$ (05)

Q.2 a) Prove that Field $\bar{F} = (z^2+2x+3y)i+(3x+2y+z)j+(y+2xz)k$ is irrotational . Find scalarpotentional $\emptyset$ such that $\bar{F} = \nabla\emptyset$ (06)

b) obtain Fourier expansion of $F(x) = 4-x^2$ in $[0,2]$ (06)

c) Using Laplace transform Solve the differential equation (08)

$y''+2y'-3y = \sin t$, Given $y(0)=0$, $y'(0) = 0$

Q.3 a) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{\sin x}{x} - \cos x\right\}$ (06)

b) Find Complex form of fourier series of $f(x) = e^{3x}$ in $[-\pi,\pi]$ (06)

c) i) Find $L^{-1}\left\{ Log\left(1+\frac{a^2}{s^2}\right)\right\}$ ii) $L^{-1}\left\{\left(\frac{e^{-2s}}{s^2+8s+25}\right)\right\}$ (08)

Q.4 a) Find analytic function f(z) whose imaginary part is $e^x(x\sin y+y\cos y)=c$ (06)

b) Find $L^{-1}\left\{\frac{s}{(s^2+4)((s^2+16)}\right\}$ by convolution theorem (06)

c) Find Fourier expansion of $f(x) = 1+\frac{2x}{\pi}$ $-\pi \le x \le 0$ (08)

$= 1-\frac{2x}{\pi}$ $0 \le x \le \pi$ Hence deduce that

$\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+$..............

Q.5 a) Find L[f(t)],Where $f(t) = 2t$, $0 < t < \pi$ (06)

$= \pi - t$, $\pi < t < 2\pi$ & $f(t + 2\pi) = f(t)$

b) Find Bilinear transformation which maps the points ∞, i,0 of Z plane onto 0, i, ∞ of W plane . also find the fixed points (06)

c) Verify Green's Theorem for $\oint (x^2 - xy^2)dx + (y^2 - 2xy)dy$ Where C is square with vertices (0 , 0) (2, 0) (0, 2) , (2, 2) (08)

Q.6 a) Using Gauss divergence Theorem , evaluate $\iint \bar{n}.\bar{F}\, ds$ Where $\bar{F} = x^3i + y^3j + z^3k$ & S is the surface bounded by sphere $x^2 + y^2 + z^2 = a^2$ (06)

b) Find image of region bounded by x=0, y=0, x = 1, y=1 in Z plane onto W plane under W = z + (2+ i) (06)

c) Find Fourier cosine integral representation of $f(x) = x$ $0 \le x \le 1$ (08)

$= 2 - x$ $1 \le x \le 2$

$= 0$ $x > 2$

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