

Mechanical/Automobile**Q P Code : NP-19673**

(3 Hours)

[Total Marks : 80]

- N.B. :**
- (1) Question No. 1 is compulsory.
 - (2) Solve any three questions out of remaining.
 - (3) Each question carries equal marks.
 - (4) Use of statistical tables is allowed.

1. (a) Find a, b, c if $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. 5
- (b) Find $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ in terms of A using Cayley-Hamilton theorem 5
for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
- (c) A continuous random variable X has the p.d.f. defined by $f(x) = A + Bx$, $0 \leq x \leq 1$. 5
if the mean of the distribution is $\frac{1}{3}$ find A and B.
- (d) A sample of 50 pieces of certain type of string was tested. The mean breaking strength turned out to be 14.5 pounds. Test whether the sample is from a batch of a string having a mean breaking strength of 15.6 pound and S.D. of 2.2 pounds. 5
2. (a) Obtain the rank correlation coefficient from the following data :— 6

X	10	12	18	18	15	40
Y	12	18	25	25	50	25
- (b) The marks of 1000 students of university are found to be normally distributed with mean 70 & SD 5. Estimate the number of students whose marks will be 6
(i) between 60 & 75 (ii) more than 75.
- (c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. 8
Find the diagonal form and transforming matrix.
3. (a) A certain injection administered to 12 patients resulted in the following changes 6
of blood pressure :
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the injection will be in general accompanied by an increase in blood pressure?
- (b) Optimize $Z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 8x_2 - 10x_3$ 6
- (c) Verify Green's theorem in the plane for 8
 $\oint (x^2 - y)dx + (2y^2 + x)dy$
around the boundary of the region defined by $y = x^2$ and $y = 4$.

4. (a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used (ii) some demand is refused. 6
- (b) Evaluate $\iint_S (\nabla \times \bar{F}) \cdot d\bar{s}$ where 6
- $\bar{F} = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$ and S is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane $Z = 9$ and open at the other end. 8
- (c) Table below shows the performances of students in Mathematics and Physics. Test the hypothesis that the performance in Mathematics is independent of performance in physics. 8

Grades in Physics	Grades in Maths		
	High	Medium	Low
High	56	71	12
Medium	47	163	38
Low	14	42	81

5. (a) The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is $\frac{1}{4}$. What is the probability of 4 successes in 6 independent trials? 6
- (b) Evaluate $\iint_S \bar{F} \cdot d\bar{s}$ where $\bar{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ and S is the region bounded by $y^2 = 4x$, $x = 1$, $z = 0$, $z = 3$. 6
- (c) Find (i) the lines of regression (ii) coefficient of correlation for the following data. 8

X	65, 66, 67, 67, 68, 69, 70, 72
Y	67, 68, 65, 66, 72, 72, 69, 71

6. (a) A group of 10 rats fed on diet A and another group of 8 rats fed on different diet B, recorded the following increase in weight
- Diet A : 5, 6, 8, 1, 12, 4, 3, 9, 6, 10gms
- Diet B : 2, 3, 6, 8, 1, 10, 2, 8 gms
- Find if the variances are significantly different? 6
- (b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \tan A = A \tan 3$. 6
- (c) Using the kuhn-Tucker conditions solve the following N.L.P.P. 8

$$\text{Maximize } Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$$

$$\text{Subject to } \begin{aligned} x_1 + x_2 &\leq 2 \\ 2x_1 + 3x_2 &\leq 12 \\ x_1, x_2 &\geq 0 \end{aligned}$$