Mechanical/Automobile 24th

IV / MECH/Prod/CBGS / NOV.16

Applied Maths- IV

QP Code :538904

Rev Course {CBGS}

[3 Hours] [Total Marks: 80

N.B (1) Question No. 1 is compulsory.

(2) Attempt any three of the remaining.

(3) Use of statistical table is allowed.

1\. (a) Using Green's theorem evaluate. 5

$\int_c \overline{F}.d\overline{r}$ where $\overline{F} = x^2\hat{i} - xy\hat{j}$ and c is the triangle having vertices A(0,2), B(2,0), C(4,2)

(b) Use Cayley – Hamilton theorem to find $2A^4-5A^3-7A+6I$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ 5

(c) If the mean of the following distribution is 16 find m,n and variance 5

X	8	12	16	20	24
P (X=x)	1/8	m	n	1/4	1/12

(d) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 Girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than girls. 5

2\. (a) Calculate Spearman's coefficient of rank correlation from the data on height and weight of 8 students 6

Height (in inches)	60	62	64	66	68	70	72	74
Weight (in lbs)	92	83	101	110	128	119	137	146

(b) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing (i) at least 2 (ii) exactly 2 (iii) at most 2 defective items in a consignment of 1000 packets using Poisson distribution. 6

(c) Find the eigen values and eigen vectors of the matrix 8

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

[Turn Over

3\. (a) Two different processes A and B are used to manufacture light bulbs. Samples were drawn from these two populations and following results were obtained 6

Population	A	B
Sample size	20	17
Sample Standard deviation	60	50

Test the hypothesis that variance of A is greater than variance of B

(b) Using the method of Lagrange's multipliers solve the following N.L.P.P. 6

Optimize $Z = 6x_1^2 + 5x_2^2$

Subject to $x_1 + 5x_2 = 7$ and $x_1, x_2 \geq 0$

(c) Prove that $\overline{F} = (2xy+z)\hat{i} + (x^2+2yz^3)\hat{j} + (3y^2z^2+x)\hat{k}$ is irrotational. Find the scalar potential for $\overline{F}$ and the work done in moving an object in this field from (1, 2, 0) to (2, 2, 1) 8

4\. (a) In an intelligence test administered to 1000 students the average score was 42 and standard deviation was 24. Find the number of students (i) exceeding the score 50 (ii) between 30 and 54 6

(b) Use Gauss's divergence theorem to evaluate $\iint_s \overline{N}.\overline{F}ds$ where $\overline{F} = 2x\hat{i} + xy\hat{j} + z\hat{k}$ over the region bounded by the cylinder $x^2+y^2 = 4$, $z = 0$, $z = 6$ 6

(c A sample of 400 students of undergraduates and 400 students of post graduate Classes was taken to know their opinion about autonomous colleges. 290 of the undergraduate and 310 of the post graduate students favored the autonomous status. Present these facts in the form of a table and test at 5% level, that the opinion regarding Autonomous status of colleges is independent of the level of classes of students 8

5\. (a) Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five? 6

(b) Use Stoke's theorem to evaluate $\int_c \overline{F}.d\overline{r}$ where $\overline{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and c is the boundary of $x=0$, $y=0$ and $x^2+y^2 = 1$ in the plane $z = 0$ 6

(c) A chemical engineer is investigating the effect of process operating temperature X on product yield Y. The results in the following data 8

X	100	110	120	130	140	150	160	170	180	190
Y	45	51	54	61	66	70	74	78	85	89

Find the equation of regression line which will be enable to predict yield on the basis of Temperature. Find also the correlation coefficient between X and Y

6. (a) Ten individuals are chosen at random from a population and their heights are found to 63, 63, 64, 65, 66, 69, 69, 70, 71, 70 inches. Discuss the suggestion that the mean height of the population is 65 inches. 6

(b) Show that the matrix A is derogatory and find its minimal polynomial 6

$$A = \begin{bmatrix} 2 & -3 & 3 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

(c) Using the Kuhn-Tucker conditions solve the following problem 8

Maximize $z = 10x_1 + 10x_2 - x_1^2 - x_2^2$

Subject to $x_1 + x_2 \leq 8$

$-x_1 + x_2 \leq 5$

$x_1, x_2 \geq 0$