S.E (MECHANICAL ENGG) (SEM III) (CBSGS) 20th November, 2014
Applied Mathematics-III 2.30 pm to 5.30 pm

Mechanical/Automobile

QP Code : 14535

(3 Hours) **[Total Marks : 80**

N.B : (1) Question no.1 is **compulsory**.
(2) Answer any **three** from remaining.
(3) **Figures** to the **right** indicate marks.

1. (a) Find laplace transform of $t^3\cos t$. 5

(b) Find the image of $|z - ai| = a$ under the transformation $w = \frac{1}{z}$. 5

(c) Construct an analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. 5

(d) Show that the set of functions $\cos nx$ $n = 1, 2, 3 \ldots$ is orthogonal on $(0, 2\pi)$. 5

2. (a) By using Convolution Theorem. Find invese laplace transform of $\frac{1}{s^2(s+1)^2}$. 6

(b) Find bilinear transformation that maps the points 2, i, -2 onto the point 1, i, -1. 6

(c) Find Fourier Series for $f(x) = \cos mx$ in $(\pi, -\pi)$ where m is not an integer. Deduce 8

that $\cos m\pi = \frac{2m}{\pi}\left(\frac{1}{2m^2} + \frac{1}{m^2-1^2} + \frac{1}{m^2-2^2} \ldots \frac{1}{m^2-n^2}\right)$ hence show that

$$\sum_1^\infty \frac{1}{9n^2-1} = \frac{1}{2} - \frac{\pi\sqrt{3}}{18}.$$

3. (a) Find Complex form of fourier series $f(x) = e^{3x}$ in $0 < x < 3$.. 6

(b) Using Crank Nicholoson method solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to $0 \le x \le 1$ $u(0, t) = 0$, 6

$u(1, t) = 0$, $u(x, 0) = 100x(1-x)$ taking $h = 0.25$ in one step.

(c) Using laplace transform solve $(D^2+2D+5)y = e^{-t}\sin t$ when $y(0) = 0$ and $y'(0) = 1$. 8

4. (a) Evaluate $\int f(z)dz$ along the Parabola $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$ where 6

$f(z) = x^2 - 2iy$

(b) Find half range cosine series for 6

$f(x) = x \quad 0 < x < \pi/2$

$= \pi - x \quad \pi/2 < x < \pi$

(c) Obtain two distinct Laurent's series of $f(z) = \frac{1}{(1+z^2)(z+2)}$ for $1 < |z| < 2$ and $|z| > 2$. 8

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5. (a) By using Bender Schmidt method solve $\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial t}$ $f(0, t) = f(5, t) = 0$. 6

$f(x, 0) = x^2 (25 - x^2)$ find f in range taking h = 1 and upto 5 seconds.

(b) Evaluate $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt$. 6

(c) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$. 8

6. (a) A string is stretched and fastened to two points distance ℓ apart. motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time t = 0. Show that the displacement of a point at a distance x from one end at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\pi\frac{ct}{\ell}\right)$. 6

(b) If $f(z) = u + iv$ is analytic and $u - v = e^x (\cos y - \sin y)$ find f(z) in terms of z. 6

(c) Evaluate : 8

$L^{-1}(2\tanh^{-1} s)$

$L^{-1}\left[\frac{s}{(s-2)^6}\right]$.