

Con. 7845-13.

Mechanical/Automobile

GX-12025

(3 Hours)

[Total Marks : 80

- N.B. :** (1) Question No. 1 is **Compulsory**.
 (2) **Answer** any **Three** from remaining.
 (3) **Figures** to the **right** indicate marks.

1. (a) Find laplace of $\sin \sqrt{t}$ 5
 (b) Show that the set of functions $\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right)$ is orthogonal over $(0, L)$. 5
 (c) Show that $u = \sin x \cos hy + 2 \cos x \sin hy + x^2 - y^2 + 4xy$ Satisfies laplace equation and find its corresponding analytic function $f(z) = u + iv$. 5
 (d) Determine constants a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. 5
2. (a) Find complex form of fourier series $f(x) = e^{3x}$ in $0 < x < 3$. 6
 (b) Using Crank Nicholson Method solve $u_t = u_{xx}$ subject to $u(x, 0) = 0$ $u(0, t) = 0$ and $u(1, t) = t$ for two time steps. 6
 (c) Solve using laplace transforms $\frac{d^2 y}{dt^2} + y = t$, $y(0) = 1$, $y'(0) = 0$. 8
3. (a) Find bilinear transformation that maps the points $0, 1 - \infty$ of the z plane into $-5, -1, 3$ of w plane. 6
 (b) By using Convolution Theorem find inverse laplace transform of $\frac{1}{(S^2 + 4S + 13)^2}$ 6
 (c) Find fourier series of $f(x) = x^2$ ($-\pi \leq x \leq \pi$) and prove that 8
 (i) $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
 (ii) $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$
 (iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
4. (a) Evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$ 6
 (b) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by 6

Bender schmidt method subject to conditions $u(0, t) = 0$ $u(x, 0) = 0$
 $u(1, t) = t$ taking $h = 0.25$ $0 < x < 1$

- (c) Obtain two distinct Laurent's Series for $f(z) = \frac{2z-3}{z^2-4z-3}$ in Powers of $(z-4)$ indicating Region of Convergence.

8

5. (a) Evaluate $\int_0^{1+i} Z^2 dZ$ along

6

(i) line $y = x$

(ii) Parabola $x = y^2$

Is line independent of path? Explian.

- (b) Find half range Cosine Series for $f(x) = e^x$ $0 < x < 1$.

- (c) Find analytic function

6

$f(z) = u + iv$ such that

8

$$u-v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^y - e^{-y}}$$

when $f(\pi/2) = 0$

6. (a) A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ in the shape defined by $y = Kx(I-x)$ where K is a Constant is released from this position of rest. Find $y(x,t)$ The vertical displacement

6

$$\text{if } \frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

- (b) Find image of region bounded by $x = 0$, $x = 2$ $y = 0$ $y = 2$ in the z plane under the transformation $w = (1+i)Z$

6

- (c) Evaluate $\int_0^{2\pi} \frac{d\theta}{25-16\cos^2\theta}$

8