(3hours)   [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = te^{-3t} \sin t$.   5

   b) Obtain Complex form of Fourier series of $f(x) = e^x$, $-1 < x < 1$ in $(-1, 1)$.   5

   c) Does there exist an analytic function whose real part is $u = k(1 + \cos\theta)$? Give justification.   5

   d) The equations of lines of regression are $3x + 2y = 26$ and $6x + y = 31$. Find i) means of x and y, ii) coefficient of correlation between x and y.   5

2. a) Evaluate $\int_0^\infty e^t \sin 2t \cos 3t \, dt$.   6

   b) Find the image of the square bounded by lines $x = 0, x = 2, y = 0, y = 2$ in the z-plane under the transformation $w = (1 + i)z + 2 - i$.   6

   c) Obtain Fourier series of $f(x) = |x|$ in $(-\pi, \pi)$. Hence, deduce that –   8
   $$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots.$$

3. a) Find the inverse Laplace transform of $F(s) = \dfrac{s}{(s^2+9)(s^2+4)}$.   6

   b) Solve $\dfrac{\partial^2 u}{\partial x^2} - 100\dfrac{\partial u}{\partial t} = 0$, with $u(0,t) = 0, u(1,t) = 0, u(x,0) = x(1-x)$

   taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender –Schmidt method. 6

   c) Using Residue theorem, evaluate

   i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ ii) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2}$ 8

[TURN OVER]

4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0,$

   $u(0,t) = 0, \ u(5,t) = 100, \ u(x,0) = 20$ taking $h = 1$ for one-time step.   6

   b) Obtain the Taylor's and Laurent series which represent the function

   $f(z) = \frac{z}{(z-1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6

   c) Solve $(D^2 - 3D + 2)y = 4e^{2t}$ with $y(0) = -3, y'(0) = 5$ where $D \equiv \frac{d}{dt}$ 8

5. a) Find an analytic function $f(z) = u + iv$, if   6
   $$u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$$

   b) Find the Laplace transform of $f(t) = t\sqrt{1 + \sin t}$ 6

   c) Obtain half range Fourier cosine series of $f(x) = x, 0 < x < 2$. Using
   Parseval's identity, deduce that –   8
   $$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

6. a) If $f(a) = \oint_C \frac{3z^2 + 7z + 1}{z - a} dz, \ C : x^2 + y^2 = 4$
   find the values of $f(3), f'(1 - i)$ and $f''(1 - i)$ 6

   b) Find the coefficient of correlation between height of father and height of son
   from the following data,   6

Height of father	65	66	67	68	69	71	73
Height of son	67	68	64	68	72	69	70

   c) A tightly stretched string with fixed end points $x = 0$ and $x = l$, in the shape
   defined by $y = kx(l - x)$ where k is a constant is released from this position
   of rest. Find $y(x,t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$.   8

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