

(3 Hours)

[ Total Marks : 80

- N.B. :** (1) Question No. 1 is compulsory.  
 (2) Attempt any three questions from the remaining five questions.  
 (3) Assume **suitable data** if needed and state it clearly.  
 (4) Figures to right indicate full marks.

1. Solve any five :

- (a) Define and explain the terms "Basis of vector space" and "subspace of a vector space". 4
- (b) Two statistically independent random variables X and Y have mean values 2 and 4 respectively. If  $W = 3X - Y$ , Find the mean and variance of random variable W. 4
- (c) State the important properties of General correlation matrices. 4
- (d) Describe application of DKLT (Discrete Karhunen-Loeve Transform) in data compression. 4
- (e) State the CRLB theorem. 4
- (f) Give assumptions about the state variable system used in Kalman filtering. 4
2. (a) Check whether the following vectors are independent  $[1 \ 3 \ 2]^T$ ,  $[2 \ 1 \ 3]^T$ ,  $[3 \ 2 \ 1]^T$ . 6
- (b) Explain the four fundamental subspaces of linear operator. 6
- (c) Explain Gram-Schmidt orthogonalization procedure. 8
3. (a) Let  $x(n) = A + w(n)$   $n=0, 1, \dots, N-1$  where  $w(n)$  is WGN with zero mean and variance  $\sigma^2$ . Determine the CRLB for A. 10
- (b) Consider a linear transformation  $y = A^T x$ . The mean vector  $\mu_x = [2 \ 1]^T$ , Find the mean vector of y if A is 2x2 identity matrix. 4
- (c) State and explain central limit theorem (CLT). 6
4. (a) Explain the concept of Innovations representation. What is whitening process. 6
- (b) Define and explain following 4
- (i) Bias of Estimator
- (ii) Efficient estimator

[TURN OVER]

- (c) Compare and contrast orthogonal and triangular decompositions for zero mean random vectors. 10
5. (a) What is Kalman filtering? Discuss in detail. 10  
 (b) A random process is defined as  $X(n) = A \cos(2\pi n)$ , where  $A$  is a Gaussian random variable with zero mean and variance  $\sigma^2$ . 10  
 (i) Determine the density function of  $X(0)$  and  $X(1)$ .  
 (ii) Is  $X(n)$  a stationary process in any sense?

6. (a) A causal LTI system is described by the difference equation 10

$$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$$

is driven by zero mean WSS process with autocorrelation  $R_x(\ell) = 2\delta(\ell)$ .

- (a) Determine the cross power spectral density between input and output  
 (b) Power spectral density at the output
- (b) It is desired to estimate the value of a DC level  $A$  in WGN or  $x(n) = A + w(n)$ ,  $n=0,1,\dots,N-1$ . 10  
 where  $w(n)$  is zero mean and uncorrelated and each sample has variance  $\sigma^2=1$ . Consider following estimator

$$\hat{A} = \frac{X(0) + X(N-1)}{2}$$

Find the mean of the estimator. Is the estimator biased? Compute the variance of the estimator.