(3 Hours) [Total Marks : 80

N.B (1) Question No1 is compulsory.
(2) Attempt any four questions out of remaining six questions.
(3) Assume any necessary data but justify the same.
(4) Figures to the right indicate full marks.
(5) Use of scientific calculator is allowed.

1. (a) i) Find the value of k such that   05
$f(x) = kx^2(1-x^3), \quad 0<x<1$
$= 0 \quad$ otherwise
Is a proper density function of a continuous variable

ii) Calculate the modal marks for the following: 05

Marks	10-30	30-50	50-70	70-90	90-110	110-130
No:of students	4	10	14	12	8	6

(b) i) The probability of occurrence of an event A is 0.7, the probability of non-   05
occurrence of B is 0.5 and non-occurrence of at least one of A and B is 0.6.
Find the probability that at least one of A and B occurs.

ii) Let X_1 and X_2 be two stochastically independent random variables having   05
variance k and 2 respectively. If variance of $Y=3X_1-X_2$ is 25 find k.

2. (a) The joint probability density function of a two dimensional random variable   08
(X,Y) is given by
$f(x,y) = 2 \quad 0<x<1, 0<y<x$
$= 0 \quad$ elsewhere

i) Find the marginal density functions of X and Y.

ii) Find the conditional density function of Y given X=x and conditional density
function of X given Y=y.

(b) Assume that the number of messages input to a communication channel in an   07
i) interval of duration t seconds is Poisson distributed with parameter 0.3 t.
Compute
the probability that exactly 3 messages will arrive during 10 seconds interval.
ii) At most 20 messages arrive in a period of 20 seconds

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3. (a) In a sample of 1000 cases ,the mean of a certain test is 14 and standard   08
        deviation is 2.5.Assuming the distribution to be normal, find
        (1)How many students scored between 12 and 15?
        (2)How many scored above 18?
        (3)How many scored below 8?
        (4)How many scored 16?
        $P(0 \leq Z \leq 0.4) = 0.1554$, $P(0 \leq Z \leq 0.8) = 0.2881$, $P(0 \leq Z \leq 1.6) = 0.4452$,
        $P(0 \leq Z \leq 2.4) = 0.4918$

    (b) In manufacturing a certain component, two types of defects are likely to occur   07
        with respective probabilities 0.05 and 0.1.What is the probability that a
        randomly chosen component has one kind of defect, given that it is found to
        be defective?

4. (a) The following are the marks obtained by 8 students in 2 subjects Computer   08
        Graphics (CG) and Probability and Statistics (PS).Calculate rank correlation
        coefficient.

Marks in CG	15	20	28	12	40	60	20	80
Marks in PS	40	30	50	30	20	10	30	60

    (b) The following table gives the number of accidents in a city during a week.   07
        Find whether the accidents are uniformly distributed over a week.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No: of accidents	10	8	11	9	12	10	10

        Given for 6 degrees of freedom at 5% level of significance the table value of
        X^2 is 12.59.

5. (a) Calculate Bowley's coefficient of skewness for the following data: 08

Wages	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No:of workers	7	12	18	8	5

    (b) A random variable X takes the values 1,2 ,3 and 4 such that   07
        $4P(X=1)= 2P(X=2)= 3P(X=3)= P(X=4)$.Find the probability distribution and
        cumulative distribution function of X.

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6. (a) i) What is the probability that 4 S's come consecutively in the arrangement of   04
the letters in the word 'MAHARASHTRA'?

ii)   04

Calculate the coefficient of variation for the following data

Daily wages(Rs)	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80
No:of workers	17	27	42	61	72	65	47	34	22	13

(b) i) Given that n=9, $\sum x^2 = 285$, $\sum y^2 = 1356$, $\sum x = 45$, $\sum y = 108$, $\sum xy = 597$.   04
Calculate correlation coefficient

ii) If hens of a certain breed lay eggs on five days of a week on an average, find   03
how many days during a season of 100 days, a poultry keeper with 5 hens of
this breed, will expect to receive at least 4 eggs?

7. (a) i) Consider discrete random variables X and Y with joint p.m.f. as below:-   04

X\Y	1	2	3
1	2/16	2/16	1/16
2	3/16	2/16	1/16
3	2/16	1/16	2/16

Find the conditional distribution of X given Y=2.

ii) Find the mean deviation about mean for the following data   04

X	10	11	12	13	14
Frequency	3	12	18	12	3

(b) i) The mean of two samples of size 50 and 100 respectively are 54.1 and 50.3   04
and standard deviation are 8 and 7 .Find the mean and standard deviation of
the sample obtained by combining the two samples.

ii) Establish the lack of memory property of geometric distribution.   03

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