Q. P. Code: 08239

(Time: $2\frac{1}{2}$ hours)

[Marks: 75]

Please check whether you have got the right question paper.

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculator is **allowed**.

1. Attempt *any three* of the following: **15**

a. What is a mathematical model? With the help of a flowchart, explain the of solving an engineering problem.

b. Create a hypothetical floating-point number set for a machine that stores information using 7-bit words. Employ the first bit for the sign of the number, the next three for the sign and the magnitude of the exponent, and the last three for the magnitude of the mantissa.

c. Suppose that you have the task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm, respectively, compute (i) the true error and (ii) the true percent relative error for each case.

d. Use zero- through fourth-order Taylor series expansions to approximate the function

$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$

from $x_i = 0$ with $h = 1$. That is, predict the function's value at $x_{i+1} = 1$.

e. Compute the condition number for

$$f(x) = tanx\ for\ \tilde{x} = \frac{\pi}{2} + 0.1\left(\frac{\pi}{2}\right)$$

$$f(x) = tanx\ for\ \tilde{x} = \frac{\pi}{2} + 0.01\left(\frac{\pi}{2}\right)$$

f. Explain blunders, formulation errors and data uncertainty.

2. Attempt *any three* of the following: **15**

a. Find the roots of the equation

$$x^3 - 12.2x^2 + 7.45x + 42 = 0$$

between 11 and 12 using Regula-Falsi method correct up to 4 decimal places.

b. Find the roots of the equation

$$xtanx = 1$$

near 4 using Newton Raphson method correct up to 4 decimal places.

c. Use the Secant method to find a solution to $x = cosx$ correct up to 4 decimal places.

d. Given log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990 and log 7 = 0.8451. Find the value of log 47.

[TURN OVER]

e. The table below gives the value of $tan\theta$. Evaluate $tan67°20'$

θ	65°	66°	67°	68°	69°
$tan\theta$	2.1445	2.2460	2.3559	2.4751	2.6051

f. From the table of Bessel function $J_n(1)$, estimate the value of $J_{\frac{3}{2}}(1)$

n	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$J_n(1)$	-0.4401	0.0447	0.4311	0.6694	0.7652	0.7522	0.6714	0.5587	0.4401

3. Attempt *any three* of the following: **15**

a. Solve the following simultaneous equations by Gauss – Jordan elimination method:

$$2x_1 + 6x_2 - x_3 = -14$$
$$5x_1 - x_2 + 2x_3 = 29$$
$$x_3 - 3x_1 - 4x_2 = 4$$

b. Solve the following simultaneous equations by Gauss – Seidel method:

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

c. For the set of points (0, 2), (2, -2), (3, -1), evaluate $\left(\frac{dy}{dx}\right)_2$

d. Evaluate $\int_0^1 \frac{1-e^{-x}}{x} dx$ using trapezoidal rule and Simpson's 3/8 rule.

e. Solve $\frac{dy}{dx} = x + y$; y(1) = 1 for the interval 1 (0.1) 1.2, using method of Taylor series.

f. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, where y(0) = 1, to find y(0.1) using Runge-Kutta method.

4. Attempt *any three* of the following: **15**

a Fit a straight line to the x and y values in the two rows:

x	1	2	3	4	5	6	7
y	0.5	2.5	2.0	4.0	3.5	6.0	5.2

b Fit a second degree parabola for the following:

x	2.5	3	3.5	4	4.5	5	5.5
y	4.32	4.83	5.27	5.47	6.26	6.79	7.23

c Fit the function $f(x; a_0, a_1) = a_0(1 - e^{-a_1 x})$ to the data:

x	0.25	0.75	1.25	1.75	2.25
y	0.28	0.57	0.68	0.74	0.79

using initial guesses $a_0 = 1$ and $a_1 = 1$. (Use Gauss Newton Method)

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d Maximize $50x+100y$ subject to $10x+5y\leq 2500$; $4x+10y\leq 2000$; $x+1.5y\leq 450$ and $x\geq 0; y\geq 0$.

e A firm makes two types of furniture – chairs and tables. The contribution for each product as calculated by the accounting department is Rs. 20 per chair and Rs. 30 per table. Both products are processed on three machines M_1, M_2 and M_3 .The time required in hours by each product and total time available in hours per week on each machine are as follows:

MACHINE	CHAIR	TABLE	AVAILABLE TIME
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

How should the manufacturer schedule his production in order maximize contribution?

f An aged person must receive 4000 units of vitamin, 50 units of minerals and 1400 calories a day. A dietician advises to thrive on two foods F1 and F2 that cost Rs 4 and Rs 2 respectively per unit of food. It one unit of F1 contains 200 units of vitamins, 1 unit of mineral and 40 calories and one unit of F2 Contains 100 units of vitamins 2 units of minerals and 40 calories, formulate a linear programming model to minimize the cost of diet.

5. Attempt *<u>any three</u>* of the following: **15**

a. The diameter of an electric cable; say X, is assumed to be a continuous random variable with p.d. f. $f(x) = 6x(1-x), 0 \leq x \leq 1$.
(i) Check that above is p.d.f,
(ii) Determine a number b such that P (X < b) = P (X> b)

b. Define and explain the concept of probability density function.

c. The probability mass function of a random variable X is zero except at the points $i = 0, 1, 2$. At these points it has the values $p(0) = 3c^3, p(1) = 4c - 10c^2, p(2) = 5c - 1$ for some $c > 0$.
(i) Determine the value of *c.*
(ii) Compute the following probabilities, $P(X < 2)$ *and* $P(1 < X \leq 2)$.
(iii) Describe the distribution function and draw its graph.
(iv) Find the largest *x* such *that* $F(x) < \frac{1}{2}$.
(v) Find the smallest *x* such that $F(x) \geq \frac{1}{3}$.

d. What is exponential distribution? Suppose the time till death after infection with Cancer, is exponentially distributed with mean equal to 8 years. If X represents the time till death after infection with Cancer, then find the percentage of people who die within five years after infection with Cancer.

e. The price for a litre of whole milk is uniformly distributed between Rs. 45 and Rs. 55 during July in Mumbai. Give the equation and graph the pdf for X, the price per litre of whole milk during July. Also determine the percent of stores that charge more than Rs. 54 per litre.

f. The monthly worldwide average number of airplane crashes of commercial airlines is 2.2. What is the probability that there will be (i) more than 2 such accidents in the next month? (ii) more than 4 such accidents in the next 2 months?
