

10/10/18

Q.P. Code: 31174

(2 1/2 Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q.1 Attempt All (Each of 5Marks)

(a) Select correct answer from the following:

(15M)

1) In which of the following method, we approximate the curve of solution by the tangent in each interval.

- a) Simpson's Method
b) Euler's method
c) Newton's method
d) None of the above

2) If $\int_0^a f(x) dx = 2 \int_0^a f(x) dx$ then $f(x)$ is called as

- a) Even function
b) odd function
c) constant function
d) none of the above

3) A function is said to be invertible if and only if it is

- a) Bijective b) injective c) Inflexion d) Surjective

4) $\lim_{x \rightarrow \infty} \frac{1}{3x}$

- a) 1 b) infinite c) zero d) None

5) A point on a curve where two curves crosses each other is called

- a) Cusps b) Asymptote c) Tangent d) Point of inflexion

(b) Fill in the blanks:

(continuous, $-\infty$, Modelling, $\frac{1}{4}$, $\frac{1}{2}$, $5x+5y-8$, $a^x \log a$)

1. $\lim_{x \rightarrow \infty} (-2x) =$ _____

2. The derivative of a^x is _____.

3. _____ is the process of writing a differential equation to describe a physical situation.

4. $\int_0^{\pi/2} \sin^2 x \cos x dx =$ _____

5. Linearization of $x^3 + xy + y^2$ at (1, 2) is _____.

(c) Answer the following in one line

1. State $\epsilon - \delta$ definition of limit
2. Define Concavity
3. Evaluate $\int [e^{2 \log x} + e^{x \log a}] dx$
4. Define the term Definite Integral
5. Define Absolute Extreme values

Q. 2 Attempt the following (Any THREE)

(15M)

- (a) Show that $\lim_{x \rightarrow 3} \frac{x^3 - 4x^2 + 13x - 30}{x - 3} = 16$
- (b) Show that $|x|$ is continuous everywhere.
- (c) Find the intervals on which function $f(x) = x^2 - 4x + 3$ is increasing or decreasing.
- (d) Find the relative extrema of $f(x) = -1 + 8x - 3x^2$ using both first and second derivative test.
- (e) Using Newton's method find the approximate root for the equation $f(x) = x^3 - x - 1$
- (f) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 72 running feet of chicken wire is available for the fence?

Q. 3 Attempt the following (Any THREE)

(15M)

- (a) Evaluate $\int 1/(9 \cos^2 x + 4 \sin^2 x) dx$
- (b) Evaluate $\int_0^{\pi} \frac{x}{(1 + \cot x)^2} dx$
- (c) Solve the differential equation $(1 + y^2) dx + (1 + x^2) dy = 0$
- (d) Using Euler's method find approximate value of y when $x = 1$, in five steps, taking $h = 0.2$ for $dy/dx = x + y$ and $y(0) = -1$.
- (e) Solve the differential equation $dy/dx + y \tan x = \cos^2 x$
- (f) Evaluate $\int_0^1 \sqrt{1 + x^2} dx$ using Simpson's rule with $n=6$

Q. 4 Attempt the following (Any THREE)

(15)

- (a) Find $\lim_{(x,y) \rightarrow (4,-2)} x(x^3 + 2y)^{1/3}$
- (b) Find the second order derivatives of $f(x,y) = y^2 e^x + y$
- (c) If $z = x^2 + y^2$, $x = a \cos t$ and $y = a \sin t$. Use chain rule to find $\frac{dz}{dt}$.
- (d) Find the directional derivative of $f(x, y, z) = x^2 y - y z^3 + z$ at the point $(1, -2, 0)$ in the direction of the vector $a = 2i + j - 2k$
- (e) Find the gradient vector of $f(x, y) = x^3 + 2xy^2$. Evaluate it at $(-3, -4)$

- (f) Find the equation for the tangent plane and parametric equations for normal line to the surface $z=x^2y$ at the point $(2, 1, 4)$

Q. 5 Attempt the following (Any THREE)

(15)

- (a) Locate all relative extrema and saddle points of

$$f(x, y) = x^3 + 2y^3 - 3x^2 - 24y + 16$$

- (b) Solve the differential equation

$$x(x+y) dy - y^2 dx = 0$$

- (c) Sketch the graph of the equation $y = x^3 + 5x + 7$ and identify the intervals where the function y is increasing and decreasing (draw the graph on the answer sheet itself)

- (d) $\int_0^{\pi/2} \sin 5x \cos 3x dx$

$$\int_0^{\pi/2} \sin 5x \cos 3x dx$$

- (e) Find the asymptotes of the function $y = \frac{1}{(x+1)(x+2)^2}$
