F.Y.CS ATKT Exam Oct-17
Sem-II
Calculus

10/10/17

(12)

Q.P. Code :05601

[Time: $2^1/_2$Hours] [Marks:75]

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 Answer following questions. 15

a) Choose the best choice for the following questions: 05

1) Let f be defined on an interval, and let x_1 and x_2 be points on the interval, then f is said to be a constant if
   p) $f(x_1)=f(x_2)=0$
   q) $f(x_1)=f(x_2)=1$
   r) $f(x_1)=f(x_2)=k$
   s) all of these

2) if f″ (a) exists and f has an inflection point at x= a, then
   p) f″ (a)> 0
   q) f″ (a)< 0
   r) f″ (a)=0
   s) none of these

3) If a function f is continuous on an interval [a,b], then which of the following is true:
   p) f is integrable on [a,b]
   q) f is differentiable on [a,b]
   r) Either (P) or (q)
   s) None of these

4) the graph of a function of two variables is a surface in
   p) 1-space
   q) 2-space
   r) 3-space
   s) None of theses

5) which of the following is true about the function f (x,y)=$\frac{xy}{1+x^2+y^2}$?
   p) Continuous everywhere
   q) Continuous except where$1+x^2+y^2=0$
   r) Either (p) or (q)
   s) Neither (p) nor (q)

b) Fill in the blanks for the following questions: 05

1. Two non-negative numbers, x and y, have a sum equal to 10. The largest possible product of the two numbers is obtained by maximizing f (x)=---------------- for x in the interval.
2. If y = f (x) is a smooth curve on the interval [a, b] then the arc length of this curve over [a,b] is defined as-----------
3. A solution of a differential equation $\frac{dy}{dx}$ - y=0 is given by --------------
4. If f (x, y)= $\sqrt{y+1}$log $(x^2$-y), the value of f (e,0) is given by---------
5. The value of lim (x, y)→ (3,2)$x\cos(\pi y)$=----------

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c) State true or false for the following questions:

1. Newtons Method uses the tangent line to y=f(x) at x= x_n to compute x_n+1.
2. The differential equation $\frac{d^2y}{dx^2} = \frac{dy}{dx}$ has a solution which is constant.
3. If f and g are functions of two variables such that f + g and f g are both continuous, then f and g are themselves continuous.
4. If f(x ,y)→L as (x, y) →(x_0,y_0) ,then f (x, y) →L as (x, y) →(x_0,y_0)along any smooth curve.
5. A function f of two variables is said to have an absolute maximum at a point (x_0,y_0) if f (x_0,y_0) ≤ f (x, y)for all points (x, y) in the domain of f.

Q.2 ANSWER ANY three of the following question: **15**

a) Find the intervals on which f(x)=x^2 -4x+3 is increasing and the intervals on which it is decreasing.

b) Use first and second derivative tests to show that f (x)=x^3-3x+3 has a relative minimum at x=1 and a relative maximum at x=-1.

c) Locate the critical points of f (x) =3x^4+12x.

d) Find the absolute maximum and minimum values of f(x) =4x^2-12x+10 in [1,2].

e) A firm determines that x units of its product can be sold daily at p Rupees per unit, where x=1000p. The cost of producing x units per day is C(x) =3000+20x.

1. Find the revenue function R(x)
2. Find the profit function P(x)
3. Assuming that the production capacity is at most 500 units per day, determine how many units the company must produce and sell each day to maximize the profit
4. Find the maximum profit.

f) The equation x^3-x-1=0 has one real solution. Approximate it by Newtons Method.

Q.3 Answer any THREE of the following questions: **15**

a) Find the area under the curve y=3$\sqrt{x}$ over the interval [1,4].

b) Find the area of the region enclosed by x=y^2 and y=x-2, integrating with respect to y.

c) Find the approximate value of $\int_1^2 \frac{1}{x}$ dx using Simpson's rule with n=14.

d) Solve differential equation $\frac{dy}{dx} = 2(1+y^2)x.$

e) Use Euler's Method with a step size of 0.5 to find approximate solution of the initial-value problem $\frac{dy}{dx} = y^{\frac{1}{3}}, y\,(x) = 1$ over 0 ≤ x≤ 4.

f) Solve the differential equation $\frac{dy}{dx} - y = e^x$ by the method of integrating factors.

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Q.4 Answer any THREE of the following question: 15

a) Find $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ (1) along x-axis and (2) along the parabola $y=x^2$.

b) Determine whether the limit exists. If so, find its value.

$$\lim_{(x,y)\to(0,0)} \frac{1-x^2-y^2}{x^2+y^2}$$

c) Find $f_x(2,1)$ and $f_y(1,2)$ for the function $f(x,y)= 10x^2y^4 - 6xy^2 + 10x^2$.

d) Find the directional derivative of $f(x, y, z) = \frac{z-x}{z+y}$ at the point (1,0,-3) in the direction of the vector $a= 6i + 3j - 2k$.

e) Find parametric equations of the tangent line to the curve of intersection of the paraboloid $z=x^2 + y^2$ And the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1,1,2).

f) Find all relative extrema and saddle points of $f(x,y)=1-x^2 - y^2$.

Q.5 Answer any THREE of the following questions: 15

a) Let $f(x)=x^2 + px + q$. Find the values of p and q such that f (1) =3 is an extreme value of f on [0,2] . Is this value a maximum or minimum?

b) Show that $y=xe^{\frac{-x^2}{2}}$ satisfies the equation xy =$(1-x^2)$y.

c) Find the area of the region below the interval [2,1] and above the curve $y=x^3$

d) Solve differential equation $\frac{dy}{dx} y = e^{2x}$.

e) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{(x^2+y^2)}}$, by converting to polar coordinates.

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