

(Time: 2½ Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.  
 2) Figures to the right indicate marks.  
 3) Illustrations, in-depth answers and diagrams will be appreciated.  
 4) Mixing of sub-questions is not allowed.

## Q. 1 Answer the following questions

(a) Choose the best choice for the following questions:

(15M)

(5M)

- (i) A function  $f$  from  $R$  to  $R$  which satisfies  $f(a) = f(b)$  implies  $a=b$  for every  $a$  and  $b$  in  $R$  is said to be  
 (a) One-to-one function (b) onto function  
 (c) Either one-to-one or onto function (d) None of these
- (ii) A relation  $R$  on a set  $X$  is such that whenever  $(x, y) \in R$ ,  $(y, x) \in R$ , then  $R$  is called  
 (a) Reflexive (b) Symmetric  
 (c) Transitive (d) None of these
- (iii) What is the coefficient of  $x^2y^2$  in the expansion of  $(x+y)^4$ :  
 (a) 4 (b) 6 (c) 8 (d) None of these
- (iv) Suppose a bookcase shelf has 5 Physics texts, 3 Chemistry texts, 6 Biology texts, and 4 Mathematics texts. Number of ways a student can choose one text of each type is given by  
 (a) 660 (b) 560 (c) 460 (d) None of these
- (v) An undirected graph with no multiple edges or loops is called  
 (a) Simple graph (b) Complex graph (c) Tree (d) Pseudo graph.

(b) Fill in the blanks for the following questions:

(5M)

- (i) A function  $f$  such that  $f(x) = x$  for any  $x$  in the domain of  $f$  is said to be a \_\_\_\_\_ function.
- (ii) A relation  $R$  on a set  $A$  is called \_\_\_\_\_ if whenever  $(a, b) \in R$ , then  $(b, a) \in R$ , for all  $a, b \in A$ .
- (iii) The Gödel number of a word  $w = a_5a_2a_3a_1a_2$  is  $2^5 3^2 5^3 7 11^2$
- (iv) The number of different license plates that can be made if each plate contains a sequence of three uppercase English letters followed by three digits is given by \_\_\_\_\_.
- (v) Let  $G$  be a directed graph and  $v$  be a vertex of  $G$ . The number of edges ending at  $v$  is called Successor



- (c) Answer the following questions: (5M)
- If the domain of the function  $f(x) = x+1$  is  $\mathbb{R}$ , what will be its co-domain?
  - Let  $S$  be a set. Determine whether there is a greatest element and a least element in the poset  $(P(S), \subseteq)$ .
  - How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?
  - Define a regular grammar.
  - What is the degree of a vertex of  $n$  undirected graph?

Q. 2 Answer any *three* of the following: (15M)

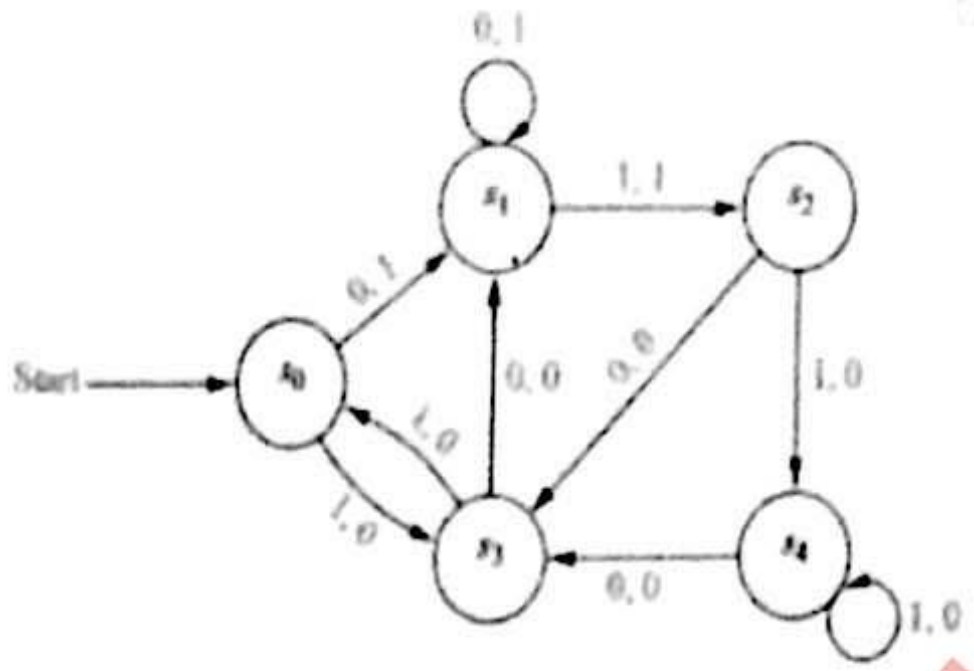
- Determine whether the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = -3x + 4$  is a bijection.
- Find the domain and range of following functions:
  - The function that assigns to each positive integer the number of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 that do not appear as decimal digits of the integer.
  - The function that assigns to a bit string the numerical position of the first 1 in the string and that assigns the value 0 to a bit string consisting of all 0s
- Draw the Hasse diagram representing the partial ordering  $\{(a,b) / a \text{ divides } b\}$  on  $\{1,2,3,4,6,8,12\}$ .
- Which of these relations on  $\{0, 1, 2, 3\}$  are partial orderings?
  - $\{(0,0),(2,2),(3,3)\}$
  - $\{(0,0),(1,1),(2,0),(2,2),(2,3),(3,3)\}$
- Find a recurrence relation and give initial conditions for the number of bit strings of length  $n$  that do not have two consecutive 0.
- Find the solution of the recurrence relation  $a_n = a_{n-1} + 2a_{n-2}$  with  $a_0 = 2$  and  $a_1 = 7$ .

Q. 3 Answer any *three* of the following: (15M)

- How many permutations of the letters ABCDEFG contain:
  - The string BCD?
  - The string CFGA?
  - The strings BA and GF?
  - The strings ABC and DE?
  - The strings ABC and CDE?
- State and prove Pascal identity.
- State Pigeonhole principle. A chess player has 77 days to prepare for an important tournament. He decides to practice by playing at least one game per day and a total of 132 games. Show that there is a succession of days during which he must have played exactly 21 games.
- Suppose that there are nine students in a discrete mathematics class at a small college.
  - Show that the class must have at least five male students or at least five female students.
  - Show that the class must have at least three male students or at least seven female students.

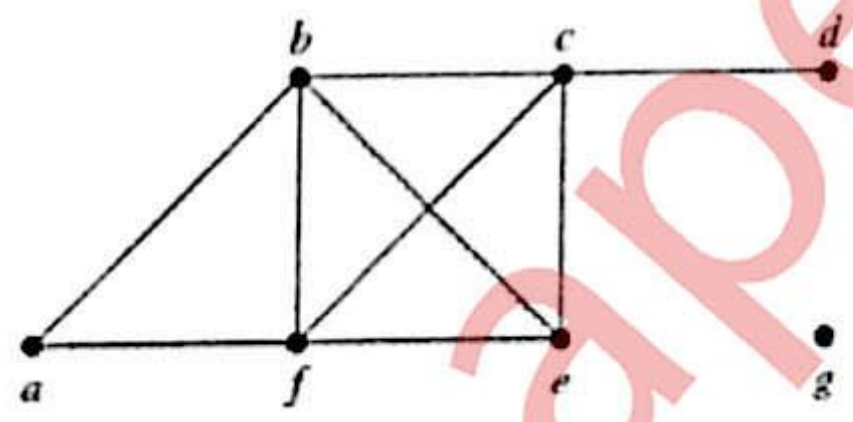


- (e) Construct a derivation tree for the following derivation:  
*the hungry rabbit eats quickly.*
- (f) Find the output string generated by the finite-state machine given below if the input string is 101011.



**Q. 4 Answer any three of the following: (15M)**

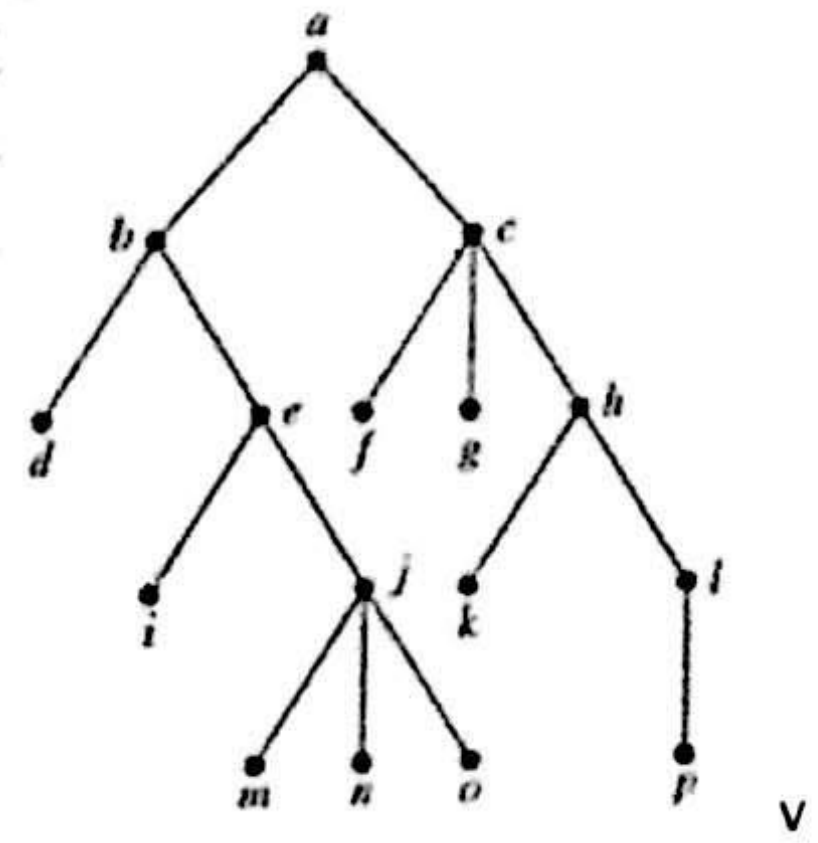
- (a) Find the degree and neighborhood of each of the vertex of the graph given below:



- (b) Suppose a graph  $G$  contains two distinct paths from a vertex  $u$  to a vertex  $v$ . Show that  $G$  has a cycle.
- (c) Draw the graph corresponding to the following adjacency matrix:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

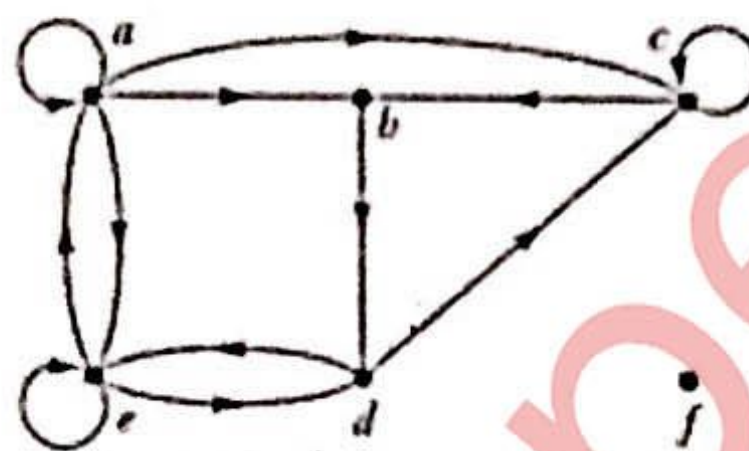
- (d) Represent the following expressions using binary tree:  
(i)  $(x + xy) + (x/y)$ ; (ii)  $x + ((xy + x)/y)$ .
- (e) Draw all possible non similar binary trees  $T$  with four external nodes.
- (f) Determine the order in which a preorder traversal visits the vertices of the following ordered rooted tree:





Q. 5 Answer any *three* of the following: (15M)

- (a) Let  $R$  be the relation on the set of all people who have visited a particular Web page such that  $xRy$  if and only if person  $x$  and person  $y$  have followed the same set of links starting at a particular Web page. Show that  $R$  is an equivalence relation.
- (b) Find the solution of the recurrence relation  $a_n = 6a_{n-1} - 9a_{n-2} - 2$  with initial conditions  $a_0 = 1$  and  $a_1 = 6$ .
- (c) What is the coefficient of  $a^{13}b^{123}$  in the expansion  $(a+b)^{25}$  using binomial theorem.
- (d) Define a language  $L$  over an alphabet  $A$ . Let  $A = \{a, b, c\}$ . Find  $L^*$  where language  $L = \{b^2\}$ .
- (e) Find the in-degree and out-degree of each vertex in the graph shown:



- (f) Consider the graph  $G$  in the following figure (where the vertices are ordered alphabetically). (i) Find the adjacency structure of  $G$ . (ii) Find the order in which the vertices of  $G$  are processed using a Breadth-first search algorithm beginning at vertex  $A$ .

