Duration – 3 Hours **Total Marks: 80**

(1) N.B.:- Question no 1 is compulsory.
(2) **Attempt any THREE questions out of remaining FIVE questions.**
(3) **Figures to the right indicate full marks.**

Q.1.a) Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+(x+\log x-x\sin y)\,dy=0$ (3)

b) Find the particular integral of $\left(D^2-2D+1\right)y=xe^x\sin x$ (3)

c) Evaluate $I=\int_0^{\pi/4}\left(1+\cos 4\theta\right)^5 d\theta$ (3)

d) Prove that $E\,\nabla=\nabla E$ (3)

e) Evaluate $\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dx\,dy\,dz$ (4)

f) Using Euler's method, find the approximate value of y, where $\frac{dy}{dx}=\frac{y-x}{\sqrt{xy}}$ with y(1) = 2 when x = 1.5 in five steps taking h = 0.1 (4)

Q.2 a) Solve $dr+(2r\cot\theta+\sin 2\theta)d\theta=0$ (6)

b) Evaluate $\int_0^\infty \frac{e^{-x}}{x}\left(1-e^{-ax}\right)dx \quad (a>-1)$ (6)

c) Change to polar and evaluate $I=\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}}\frac{dxdy}{\sqrt{(a^2-x^2-y^2)}}$ (8)

Q.3 a) Evaluate $I=\int_0^1 x^4\cos^{-1}x\,dx$ (6)

b) Evaluate $\iiint\frac{dxdydz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2+y^2+z^2=a^2$ (6)

c) Apply method of variation of parameter to solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{-x}\log x$ (8)

Q. 4 a) Find the mass of a plate in the form of a cardioid $r = a(1-\cos\theta)$, if the density at any point of the plate varies as its distance from the pole. (6)

b) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$ (6)

c) Using fourth order Runge-Kutta method, solve numerically, the differential equation $\frac{dy}{dx} = xy$ with the given condition y(1) =2, find y at x=1.2,1.4 (8)

Q. 5 a) Evaluate $\iint xy\, dxdy$ over the region bounded by $x^2 + y^2 - 2x = 0,\ y^2 = 2x \text{ and } y = x$ (6)

b) A resistance of $100\ \Omega$ and inductance of 0.5 H are connected in series with a battery of 20 V. Find the current at any instant if the relation between L, R, E is $L\frac{di}{dt} + Ri = E$ (6)

c) Evaluate $\int_0^1 \frac{dx}{1+x}$ by using (i) Trapezoidal Rule, (ii) Simpson's $(1/3)^{rd}$ Rule and (iii) Simpson's $(3/8)^{th}$ Rule. Compare the result with exact solution. (8)

Q. 6 a) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ (6)

b) Show that the length of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is $a\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$ (6)

c) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$ (8)

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