Q. P. Code: 16067

Duration – 3 Hours **Total Marks: 80**

(1) N.B.:- Question no 1 is compulsory.

(2) Attempt any THREE questions out of remaining FIVE questions.

K. G. C. E. EXAM * KARJAT, RAIGAD *

1) a) Solve $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ (4)

b) *Solve* $\frac{d^4y}{dx^4}+5\frac{d^2y}{dx^2}-36y=0$ (3)

c) Evaluate $\int_0^\infty e^{-x^4}dx$ (3)

d) Express the following integral in polar co-ordinates (4)

$$I=\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} f(x,y)dxdy$$

e) Prove that Prove that $\left(\frac{E^4-1}{\Delta}\right)y_0 = y_0 + y_1 + y_2 + y_3$. (3)

f) Evaluate $I=\int_0^{\pi/2}\int_{\pi/2}^{\pi}\cos(x+y)dx\,dy$ (3)

2 a) Solve $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sin y$. (6)

b) Change the order of integration and evaluate $I = \int_0^1 \int_{4y}^4 e^{x^2}\,dxdy$ (6)

c) Evaluate $\int_0^\pi \frac{dx}{a+b\cos x}$ $a.>0,$ $|b|<a$ and hence deduce that (8)

$$\int_0^\pi \frac{dx}{(a+b\cos x)^2}=\frac{\pi a}{\left(a^2-b^2\right)^{3/2}} \text{ and } \int_0^\pi \frac{\cos x dx}{(a+b\cos x)^2}=\frac{-\pi a}{\left(a^2-b^2\right)^{3/2}}$$

3 a) Evaluate $I=\int_0^{\log 2}\int_0^x\int_0^{x+y} e^{x+y+z}dxdydz$ (6)

b) The density at any point of a cardioide $r=a(1+\cos\theta)$ varies as the square of its distance from its axis of symmetry. Find its mass. (6)

c) Solve $(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=6x$ (8)

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4 a) Show that the length of the arc of the curve $y=\log\left(\frac{e^x-1}{e^x+1}\right)$ from x=1 to x=2 is $\log\left(e+\frac{1}{e}\right)$ (6)

b) Solve $(D^3-2D^2+D)y=x^2+x$ (6)

c) Use Runge-Kutta method of fourth order to compute $y(0.1)$ & $y(0.2)$, given $y'=xy+y^2, y(0)=1$ (8)

5 a) Use method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\frac{1}{1+\sin x}$ (6)

b) Using Taylor's series method, find $y(1.1)$ correct to four decimal places, given $y'=xy^{1/3}$ and $y(1)=1$. (6)

c) Find the value of the integral $\int_0^1 \frac{x^2}{1+x^3}dx$ by taking h= 0.2 , using (8)

(i) Trapezoidal Rule (ii) Simpson's 1/3 Rule.

Compare the errors with the exact value of the integral

6 a) The equation of an L-R circuit is given by $L\frac{di}{dt}+Ri=10\sin t$, if i = 0 at t = 0, express i as a function of t. (6)

b) Evaluate $\iiint(x^2y^2+y^2z^2+z^2x^2)dxdydz$ over the volume of the sphere $x^2+y^2+z^2=a^2$ (6)

c) Find the volume cut off from the paraboloid $x^2+\frac{1}{4}y^2+z=1$ by the plane $z=0$. (8)

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